Introduction to algebraic topology, Spring 2013

Homework 7, due Tuesday, March 12

1. Explain why the long exact sequence of a pair (X, A) is exact in the $\pi_n(A)$-term.

2. What does the long exact sequence of a pair tell you in the case (a) $A = X$, (b) A is contractible.

3. Write down the sequence for (D^2, S^1), where S^1 is the boundary of the disk D^2. Determine relative homotopy groups $\pi_n(D^2, S^1)$.

4. Use the long exact sequence of a fibration to reprove the result that, for a covering $p : \widetilde{X} \longrightarrow X$, the induced map on π_n is an isomorphism for $n \geq 2$ (hint: specialize to the case of path-connected X).

5. (a) Using the exact sequence for the Hopf fiber bundle with total space S^{2n+1}, base complex projective space $\mathbf{CP}^n$, and fiber S^1 determine homotopy groups $\pi_i(\mathbf{CP}^n)$ for $i \leq 2n$.
(b) By taking n to infinity, we get a fiber bundle with contractible total space S^∞, fiber S^1 and infinite complex projective space $\mathbf{CP}^\infty$ as the base. Find homotopy groups $\pi_i(\mathbf{CP}^\infty)$. How many of them are non-zero?

6. We proved in class that any fiber bundle over the closed interval $[0, 1]$ is trivial. Apply this result and ideas from its proof to show that any fiber bundle over $\mathbb{R}$ is trivial.