**Introduction to algebraic topology, Spring 2013**

**Homework 6, due Tuesday, March 5**

1. List all regular coverings among 14 coverings depicted on page 58 of Hatcher. Which of these coverings have infinite degree?
Give an example of a degree 4 covering of $S^1 \vee S^1$ which is not path-connected.

2. Prove that $\pi_n(X \times Y) \cong \pi_n(X) \times \pi_n(Y)$ for any topological spaces $X, Y$ (since the groups are abelian for $n > 1$, we can also write the right hand side as $\pi_n(X) \oplus \pi_n(Y)$).

3. Show that if a CW-complex $X$ has no cells of dimension $1, 2, \ldots, n$ then $\pi_n(X) = 0$ for any $i \leq n$. In particular, $\pi_i(S^n) = 0$ if $i < n$.

4. Compute the second fundamental group of the Möbius band and the Klein bottle.

5. If $X$ is a graph, then $\pi_n(X) = 0$ for all $n \geq 2$.

6. Show that the Möbius band is the total space of a fibration with the interval $I = [0, 1]$ as fiber and circle as the base.

7. Explain why the Hopf fibration $p : S^3 \times S^2$ is non-trivial (not isomorphic to the projection from the direct product $S^2 \times S^1$ onto $S^2$).