Introduction to algebraic topology, Spring 2013

Homework 5, due Tuesday, February 26

1. Mark the squares that are followed by correct statements.

☐ Any CW-complex is path-connected.

☐ Any topological space has a universal covering space.

☐ Any locally simply-connected space is locally path-connected.

☐ Any locally path-connected space is path-connected.

☐ Fundamental group of the 3-sphere S^3 is abelian.

☐ Fundamental group of the bouquiet $S^1 \vee S^2$ is finite.

☐ Any CW-complex with one 0-cell, two 1-cells, and one 2-cell has infinite fundamental group.

☐ Any covering of S^1 is a compact topological space.

☐ A connected CW-complex with no 1-cells is simply-connected.

☐ Direct product of two locally path-connected spaces is locally path-connected.

2. For each topological space X below, write next to it the universal covering space of X and the fundamental group $\pi_1(X)$.

$\mathbb{RP}^3$

$\mathbb{R}$

$[0,1]$

$\mathbb{RP}^1$

Möbius band

Torus T^2

$\mathbb{RP}^2 \times S^3$.

3. For a covering space $p : \widetilde{X} \longrightarrow X$ and a subspace $A \subset X$, let $\widetilde{A} = p^{-1}(A)$. Show that the restriction $p : \widetilde{A} \longrightarrow A$ is a covering space.

4. Some covering spaces of $S^1 \vee S^1$ are depicted on page 59 of Hatcher (pdf page 67). Draw an example of a degree 5 covering space of $S^1 \vee S^1$. Choose a base point and describe the corresponding

subgroup of F_2. Is your covering regular? Can you give an example of an irregular covering of degree 5?

5. Construct a simply-connected (universal) covering space of the subspace $X \subset \mathbb{R}^3$ that is the union of a sphere and a diameter.

6. Describe the universal covering space of $\mathbb{RP}^2 \vee S^2$.

Extra credit:

I. Let $p : \widetilde{X} \longrightarrow X$ be a covering space with $p^{-1}(x)$ finite and nonempty for all $x \in X$. Show that $\widetilde{X}$ is compact Hausdorff iff X is compact Hausdorff.

II. Exercise 6 from Hatcher Section 1.3 (page 79).