Introduction to algebraic topology, Spring 2013

Homework 3, due Tuesday, February 12

1. (20 points) Describe a triangulation of (a) $\mathbb{RP}^2$, (b) Klein bottle, (c) 3-dimensional cube $I^3 = I \times I \times I$. In each case check whether your triangulation is a simplicial complex. If it's not, explain how to refine your triangulation to turn it into a simplicial complex.

2. (15 points) Form a triangulation of the square with two 2-dimensional simplices by drawing a diagonal. Classify simplicial maps from this triangulation to the 1-simplex.

3. (10 points) The boundary of an $(n+1)$-simplex is a simplicial decomposition K of the n-dimensional sphere. Determine the number of n-simplices in the second barycentric subdivision $K^{(2)}$ of K.

4. (15 points) Describe a triangulation of the two-plane $\mathbb{R}^2$. Can you modify your construction to get a triangulation of punctured $\mathbb{R}^2$ (two-plane minus a point)?

5. (15 points) Which of the following spaces admit a triangulation?

(a) $\mathbb{C} \times I$ - complex numbers with the usual topology times the interval.
(b) $\mathbb{Q}$ - rational numbers with the topology induced from $\mathbb{R}$.
(c) $(0, 1]$ - semiopen interval.
(d) $\mathbb{Z}$ - integers with the discrete topology.
(e) $S^3 \setminus \{p_1, p_2, p_3\}$ - the three-dimensional sphere with three points removed.
(f) $S^3 \vee T^2$ - the bouquiet of the 3-sphere and 2-dimensional torus (2-torus).
(g) SX - suspension of X, if X admits a triangulation.

6. (10 points) Show that any finite simplicial complex can be embedded into the Euclidean space in such a way that the embedding is linear on each simplex.