Introduction to algebraic topology, Spring 2013

Homework 2, due Tuesday, February 5

1. Classify all CW-complexes with two 0-cells and two 1-cells up to (a) homeomorphism, (b) homotopy equivalence.

2. Describe a CW-complex homeomorphic to $\mathbb{R}^2$. Can you generalize your construction to $\mathbb{R}^n$? Also describe a CW-complex homeomorphic to punctured $\mathbb{R}^2$.

3. Show that punctured $\mathbb{RP}^n$ is homotopy equivalent to $\mathbb{RP}^{n-1}$.

4. Construct a 2-dimensional cell complex that contains both an annulus $S^1 \times I$ and a Möbius band as deformation retracts.

5. Which of the following pairs (space, subspace) satisfy the homotopy extension property?

(a) (S^n, p), where p is a point in the n-sphere.
(b) $(D^n, \partial D^n)$ - an n-disk and its boundary.
(c) $([0,1], (0,1])$ - closed interval and semiopen interval.
(d) $(\mathbb{R}, \mathbb{Z})$ - real numbers and its subset of integers.
(e) $(\mathbb{R}, \mathbb{Q})$ - real numbers and its subset of rationals.
(f) (CX, X) - cone over X and X itself.

6. Show that a suspension of any finite connected graph is homotopy equivalent to a bouquiet of 2-spheres. What can you say about suspension of finite graphs that are not necessarily connected?

(Exercise 4 is from Hatcher, Chapter 0).