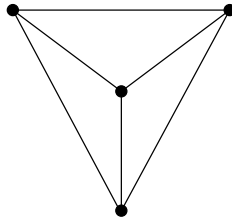


## Introduction to algebraic topology, Spring 2013

### Homework 12, due Thursday, May 2

Read Hatcher Section 2.C (Lefschetz fixed point theorem, pages 179–182)

- (a) Suppose that a map  $f : S^1 \rightarrow S^1$  has winding number 3. What is the Lefschetz number of  $f$ ? Does  $f$  necessarily have a fixed point?  
(b) Exercise 2 in Hatcher, end of Chapter 2, page 184.
2. Consider the graph  $Y$  below.



Suppose that  $f : Y \rightarrow Y$  is homotopic to the map  $g$  which is given by rotating  $Y$  clockwise 120 degrees about its central vertex. What is the Lefschetz number of  $f$ ? Does any such  $f$  have a fixed point?

3. What are the homology groups of the graph  $Y$  from problem 2 with coefficients (a) in  $\mathbb{Q}$ , (b) in  $\mathbb{Z}/9$ ?
4. Determine homology groups of the Klein bottle  $KB$  with coefficients in  $\mathbb{Z}/2$ ,  $\mathbb{Z}/3$ ,  $\mathbb{Z}/4$ , and  $\mathbb{Q}$ . (Hint: choose a simple cell decomposition of  $KB$ )
5. Determine homology groups of (a)  $\mathbb{R}P^3$ , (b) the 2-torus  $T^2$  with coefficients in  $\mathbb{Z}/2$ ,  $\mathbb{Z}/8$ , and  $\mathbb{R}$ .

**Extra credit:** Exercises 4 and 7 in Hatcher, end of Section 2, page 184.