Introduction to algebraic topology, Spring 2013

Homework 11, due Tuesday, April 23

Exercises from Hatcher Section 2.2 (pages 155-157) 4, 9ac, 12, 20.

1. (a) We construct a 2-dimensional CW complex X by starting with the 1-skeleton $X^1 = S^1 \vee S^1$ a bouquiet of two circles a, b and adding two 2-cells via attaching maps taking the boundaries to words $a^2ba^2b^{-1}$ and $a^{-1}b$. Determine the homology of X.

(b) Same question as in (a) but now we attach three 2-cells via maps given by words $ab^2a^{-1}, bababa$, and aba^2.

2. We glue a 3-disk to S^2 via an attaching map of degree 5. What is the homology and the Euler characteristic of the resulting CW-complex?

3. Compute homology and the Euler characteristic of the bouquiet $S \vee \mathbb{RP}^2$, where S is S^2 with attached equatorial disk.

4. Determine homology and the Euler characteristic of the unoriented surface N_n which is the connected sum of n copies of $\mathbb{RP}^2$:

$$N_n = \mathbb{RP}^2 \# \mathbb{RP}^2 \# \ldots \# \mathbb{RP}^2.$$