

① a.) $\|v\| = \sqrt{4+1+1} = \sqrt{6}$ so $\vec{e}_v = \frac{1}{\sqrt{6}}(2, -1, 1)$

b.) $\|\text{proj}_v \vec{u}\| = \vec{u} \cdot \vec{e}_v = (4, 4, 5) \cdot (2, -1, 1) \frac{1}{\sqrt{6}} = \frac{9}{\sqrt{6}}$ or $\frac{3}{2}\sqrt{6}$

c.) $\vec{u}_{||} = \frac{\vec{u} \cdot v}{v \cdot v} v = \frac{9}{6}(2, -1, 1) = (3, -\frac{3}{2}, \frac{3}{2})$

$\vec{u}_{\perp} = \vec{u} - \vec{u}_{||} = (4, 4, 5) - (3, -\frac{3}{2}, \frac{3}{2}) = (1, \frac{11}{2}, \frac{7}{2})$

② let $\vec{u} = \vec{AB} = (3, 1, 3)$ and $\vec{v} = \vec{AC} = (3, 0, 6)$

a.) $u \times v = \begin{vmatrix} i & j & k \\ 3 & 1 & 3 \\ 3 & 0 & 6 \end{vmatrix} = (6, -9, -3) \neq \vec{0}$ so not collinear

b.) $\|u \times v\| = \sqrt{36+81+9} = \sqrt{126}$ so $A_{\Delta} = \frac{1}{2}\sqrt{126} = \frac{3}{2}\sqrt{14}$

c.) $(6, -9, -3) \cdot (x+2, y-1, z+1) = 6x - 9y - 3z + 18 = 0$
ie. $2x - 3y - z + 6 = 0$

③ a.) $\ell(t) = (1, 0, 1) + t(-4, 2, 2) = \begin{matrix} (1-4t, & 2t, & 1+2t) \\ x & y & z \end{matrix}$

b.) $\frac{x-1}{-4} = \frac{y}{2} = \frac{z-1}{2}$ or $x-1 = -2y = -2z+2$

c.) $r^2 = 1^2 + 0^2 = 1$ so $r = 1$
 $\tan \theta = \frac{0}{1} = 0$ so $\theta = 0$ or π } $(r, \theta, z) = (1, 0, 1)$

d.) $\rho^2 = 1^2 + 0^2 + 1^2 = 2$ so $\rho = \sqrt{2}$
 $1 = \sqrt{2} \cos \varphi \Rightarrow \cos \varphi = \frac{1}{\sqrt{2}}$ so $\varphi = \frac{\pi}{4}$ } $(\rho, \theta, \varphi) = (\sqrt{2}, 0, \frac{\pi}{4})$

④ a.) $\vec{n}_1 = (1, -1, 0)$ and $\vec{n}_2 = (0, -1, 1)$

$\vec{n}_1 \cdot \vec{n}_2 = (1, -1, 0) \cdot (0, -1, 1) = 1 = \|\vec{n}_1\| \|\vec{n}_2\| \cos \theta = \sqrt{2}\sqrt{2} \cos \theta$
so $\cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

b.) $\vec{n} = (1, 2, 4) \Rightarrow (1, 2, 4) \cdot (x-1, y-2, z+1) = 0$
 $x+2y+4z-1=0$

Problem 5. For each equation below, find the surface in \mathbf{R}^3 that matches it.

(a) 8 $x^2 + 4y^2 + 4z^2 = 16$

(b) 9 $4x^2 + y^2 + 4z^2 = 16$

(c) 5 $z = 9x^2 + 4y^2$

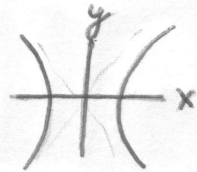
(d) 6 $z = 9x^2 - 4y^2$

(e) 1 $9x^2 + 4y^2 = 2z^2 + 72$

(f) 3 $9x^2 + 4z^2 = 2y^2 - 72$

(g) 4 $9x^2 + 4y^2 = 2z^2$

(h) 11 $9x^2 - 4y^2 = 72$ (hyperbolic paraboloid)



~~scribble~~

