Math 233 Fall 2008 Exam 1 Solutions

① a.) $\|v\| = \sqrt{4+1+1} = \sqrt{6}$ so $\vec{e}_v = \frac{1}{\sqrt{6}}(2,-1,1)$

b.) $\|\text{proj}_v \vec{u}\| = \vec{u} \cdot \vec{e}_v = (4,4,5)\cdot(2,-1,1)\frac{1}{\sqrt{6}} = \frac{9}{\sqrt{6}}$ or $\frac{3}{2}\sqrt{6}$

c.) $\vec{u}_{\parallel} = \frac{u \cdot v}{v \cdot v} v = \frac{9}{6}(2,-1,1) = (3, -\frac{3}{2}, \frac{3}{2})$

$\vec{u}_{\perp} = \vec{u} - \vec{u}_{\parallel} = (4,4,5) - (3, -\frac{3}{2}, \frac{3}{2}) = (1, \frac{11}{2}, \frac{7}{2})$

② let $\vec{u} = \vec{AB} = (3,1,3)$ and $\vec{v} = \vec{AC} = (3,0,6)$

a.) $u \times v = \begin{vmatrix} i & j & k \\ 3 & 1 & 3 \\ 3 & 0 & 6 \end{vmatrix} = (6,-9,-3) \neq \vec{0}$ so not collinear

b.) $\|u \times v\| = \sqrt{36+81+9} = \sqrt{126}$ so $A_{\Delta} = \frac{1}{2}\sqrt{126} = \frac{3}{2}\sqrt{14}$

c.) $(6,-9,-3)\cdot(x+2, y-1, z+1) = 6x - 9y - 3z + 18 = 0$

ie. $2x - 3y - z + 6 = 0$

③ a.) $\ell(t) = (1,0,1) + t(-4,2,2) = (\underset{x}{1-4t}, \underset{y}{2t}, \underset{z}{1+2t})$

b.) $\frac{x-1}{-4} = \frac{y}{2} = \frac{z-1}{2}$ or $x - 1 = -2y = -2z + 2$

c.) $r^2 = 1^2 + 0^2 = 1$ so $r = 1$

$\tan\theta = \frac{0}{1} = 0$ so $\theta = 0$ or $\cancel{\pi}$

$(r, \theta, z) = (1, 0, 1)$

d.) $\rho^2 = 1^2 + 0^2 + 1^2 = 2$ so $\rho = \sqrt{2}$

$1 = \sqrt{2}\cos\varphi \Rightarrow \cos\varphi = \frac{1}{\sqrt{2}}$ so $\varphi = \frac{\pi}{4}$

$(\rho, \theta, \varphi) = (\sqrt{2}, 0, \frac{\pi}{4})$

④ a.) $\vec{n}_1 = (1,-1,0)$ and $\vec{n}_2 = (0,-1,1)$

$\vec{n}_1 \cdot \vec{n}_2 = (1,-1,0)\cdot(0,-1,1) = 1 = \|\vec{n}_1\|\,\|\vec{n}_2\|\cos\theta = \sqrt{2}\sqrt{2}\cos\theta$

so $\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

b.) $\vec{n} = (1,2,4) \Rightarrow (1,2,4)\cdot(x-1, y-2, z+1) = 0$

$x + 2y + 4z - 1 = 0$

Problem 5. For each equation below, find the surface in $\mathbf{R}^3$ that matches it.

(a) __8__ $x^2 + 4y^2 + 4z^2 = 16$

(b) __9__ $4x^2 + y^2 + 4z^2 = 16$

(c) __5__ $z = 9x^2 + 4y^2$

(d) __6__ $z = 9x^2 - 4y^2$

(e) __1__ $9x^2 + 4y^2 = 2z^2 + 72$

(f) __3__ $9x^2 + 4z^2 = 2y^2 - 72$

(g) __4__ $9x^2 + 4y^2 = 2z^2$

(h) __11__ $9x^2 - 4y^2 = 72$ (turned axes)