ABSTRACT

Topics in Stochastic Control with Discretionary Stopping

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This thesis addresses two stochastic control problems with discretionary stopping. The first one is a finite-fuel, singular stochastic control problem of optimally tracking the standard Brownian motion $x + W(\cdot)$ started at $x \in \mathbb{R}$, by an adapted process $\xi(\cdot) = \xi^+(\cdot) - \xi^-(\cdot)$ of bounded total variation $\check{\xi}(t) \doteq \xi^+(t) + \xi^-(t) \leq y, \ \forall\, 0 \leq t < \infty$, so as to minimize the total expected discounted cost

$$\mathsf{E}\left[\int_0^\tau e^{-\alpha t}\lambda X^2(t)\,dt + \int_{[0,\tau]} e^{-\alpha t}\,d\check{\xi}(t) + e^{-\alpha\tau}\delta X^2(\tau)\cdot 1_{\{\tau<\infty\}}\right]$$

over such processes $\xi(\cdot)$ and stopping times τ. Here $X(\cdot) = x + W(\cdot) + \xi(\cdot)$, and $\alpha > 0$, $\delta \geq 0$, $\lambda > 0$ are given real numbers. In its form $\delta = 0$, $\tau \equiv \infty$, this problem goes back to the seminal paper of Beneš, Shepp & Witsenhausen (1980). For fixed $\alpha > 0$ and $\delta > 0$ we characterize explicitly the optimal policy in the case $\lambda \geq \alpha\delta$ (of the "act–or–stop" type, since the continuation cost is relatively large), and in the case $0 < \lambda \leq \lambda^*$ with

$$\lambda^* \doteq \frac{\alpha\delta}{1 + \frac{\delta/\alpha}{\frac{1}{4\delta} + \frac{1}{\sqrt{2\alpha}}}}$$

(of the "act, stop, or wait" type, since the relative continuation cost is relatively small). In the latter case, an associated free-boundary problem is solved exactly.

The case $\lambda^* < \lambda < \alpha\delta$, of "moderate" relative continuation cost, is suggested as an open question.

The second problem is a utility maximization problem of mixed *optimal stopping/control* type, which can be solved by reduction to a family of related *pure optimal stopping* problems. Sufficient conditions for the existence of optimal strategies are provided in the context of continuous-time, Itô process models for complete markets. The mathematical tools used are those of optimal stopping theory, continuous-time martingales, convex analysis and duality theory. Several examples are solved explicitly, including one which demonstrates that optimal strategies need not always exist.