

Columbia University in the City of New York | New York, N.Y. 10027

DEPARTMENT OF MATHEMATICS
DEPARTMENT OF STATISTICS

508 Mathematics Building
618 Mathematics Building
2990 Broadway

Fall Semester 2007

Professor Ioannis Karatzas

G7260: TOPICS IN STOCHASTIC ANALYSIS

Tue-Thu 2:40-3:55 Room 1025 SSW Bldg

Course Description

Prerequisites: A course on Real Analysis and Probability, at the level of G4151- 4153 or G6105-6106.

Overview of Stochastic Analysis: Elements of the general theory of random processes. Martingales. Doob-Meyer decomposition. Square-integrable, Local- and Semi-Martingales. The Brownian Motion process, connections with partial differential equations. The Poisson problem, stochastic representation of its solution.

Integration with respect to continuous semi-martingales, Ito's change-of-variable formula, Girsanov's theorem and its applications. The martingale representation property for Simple Random Walk and for the Brownian filtration, the Clark-Ocone formula.

Local Time: Definition, properties, the Ray-Knight theorems.

Bessel processes: general properties, Pitman's $2M-X$ property, the D. Williams decomposition.

Stochastic Differential Equations and Diffusion Processes: The Ito-Doebelin theory of diffusions. Existence and Uniqueness results. The martingale problem of Stroock and Varadhan. Linear diffusions. Connections with partial differential equations.

Optimal Stopping and Dynamic Allocation problems. *Stochastic Control* problems: the general probabilistic theory, examples. *Filtering* problems: the Kalman-Bucy filter, enlargements of filtrations.

Mathematical Finance: Notions and problems of Hedging, Portfolio Optimization, Diversity, Arbitrage, Completeness.

Special Topics (as time permits): Least-squares approximation of random variables by stochastic integrals; Levy processes; Hypercontractivity; Skorohod embedding; Stochastic Differential Geometry; Excursions.

RECOMMENDED TEXTS:

I.KARATZAS & S.E.SHREVE (1987) *Brownian Motion and Stochastic Calculus.* Springer-Verlag, New York.

L.C.G.ROGERS & D.WILLIAMS (1987) *Diffusions, Markov Processes and Martingales, Vol. II: Ito Calculus.* J. Wiley & Sons, Chichester and New York.

D.REVUZ & M.YOR (1991) *Continuous Martingales and Brownian Motion.* Springer-Verlag, New York.

REMARKS:

My intention is to run this course as a seminar, with occasional student presentations on Thursdays. Most PhD students should want to take this course Pass-Fail. Students who wish to register and need a letter grade, should contact me ahead of time; for those students a *Take-Home Final* will be available at the end of the semester.