Intro Modern Algebra I HW5 Solution

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Problem 1

1. (a) (1 2 4 5 3)
   (b) (1 4)(3 5)
   (c) (1 3)(2 5)
   (d) (2 4)
2. (a) (1 3 5)(2 4)
   (b) (2 5 3)
   (c) (1 4)(2 3)
   (d) (1 2)(5 6)
   (f) (1 3 2 5 4)
   (j) e
   (m) (1 2)

Problem 2

Every element in S_7 can be written as a product of disjoint cycles, each of order between 2 and 7. The order of a product of disjoint cycles is the lcm of the order of the cycles, and the cycles can sum up to less than or equal 7. Therefore the possible orders of elements in S_7 are: 1, 2(=lcm(2,2)=lcm(2,2,2)), 3(=lcm(3,3)), 4(=lcm(2,4)), 5, 6(=lcm(2,3)=lcm(2,2,3)), 7, 10=lcm(2,5), 12=lcm(3,4).

An element of maximal order would be the product of a 3-cycle and a 4-cycle, an example would be (1 2 3)(4 5 6 7).

Problem 3

(1 2)(2 3)=(1 2 3) but (2 3)(1 2)=(1 3 2).

Problem 4

- The identity map is the identity map in D.
- If two permutations preserve the adjacent corners of the pentagon, then their composition would also preserve the adjacent corners since for each vertex, its set of adjacent vertices remain the same.
- The inverse map of a permutation that sends adjacent vertices to adjacent vertices would also send adjacent vertices to adjacent vertices, since otherwise their composition would not preserve adjacent vertices and cannot be the identity.

Therefore, it is clear that D is a subgroup of S_5. In particular, notice that any permutation in D is a symmetry of the pentagon, and any symmetry of the pentagon also preserves adjacent corners thus lies in D. Therefore, $D = D_5$ and its order is 10.

Problem 5

(a) 3 because it is a 3-cycle

(b) 3 because it is the product of two disjoint 3-cycles

(c) 5 because (1 4 2 3 5)^2=(1 2 5 4 3) is a 5-cycle.