## MODERN ALGEBRA I GU4041

## Homework 5, due October 12: Permutations

1. Judson, Section 5.4, exercise 1 and 2 (a)-(d), 2(f), 2(j), 2(m).
2. Let $S_{n}$ denote the group of permutations of $n$ letters. List the possible orders of all elements of $S_{7}$ and exhibit an element whose order is maximal and write it as the product of disjoint cycles.
3. Find two permutations of 4 letters $\sigma$ and $\tau$ such that $\sigma^{2}=\tau^{2}=e$ but $\sigma \tau \neq \tau \sigma$.
4. Draw a pentagon and label its corners $1,2,3,4,5$. Let $D \subset S_{5}$ be the set of permutations of the corners that take adjacent corners to adjacent corners. Show that $D$ is a subgroup of $S_{5}$. What is its order?
5. What are the orders of the following permutations?
(a) (231) in $S_{3}(\mathrm{~b})(165)(234)$ in $S_{6}$ (c) $(14235)^{2}$ in $S_{5}$.

## Recommended Reading

Judson book, Section 5.1; Howie's notes, Chapter 4.

