Mechanical Mathematicians

Duke University PLUM Lecture

Michael Harris

Columbia University

March 28, 2019

Mechanical Mathematicians

Michael Harris

03/28/2019

LSRC B101

4:30-5:30pm

The Antikythera mechanism, discovered in 1901 and dated to some time between 200 BC and 87 BC, used as an analogue computer to predict astronomical events.

It is on display in the National Museum of Archeology in Athens.

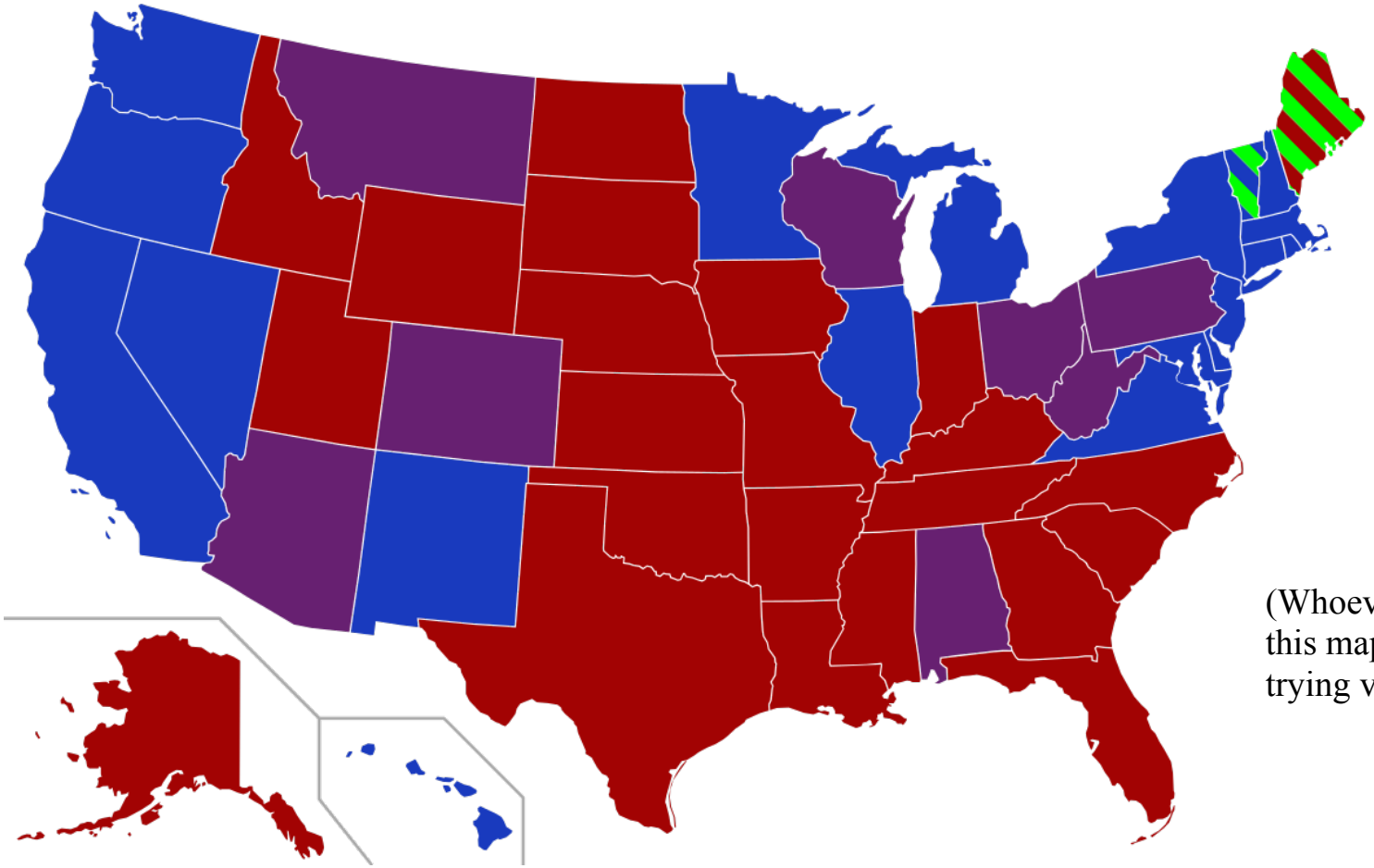
PLUM

Mathematics @ Duke University

PLUM is supported in part by the generosity of Goh Peng Ooi

Outline

Three colors don’t suffice



(Whoever colored this map wasn't trying very hard!)

Four colors suffice: Proof completed by computer

The Four-Color Theorem: for any map no more than 4 colors are required so that no two adjacent regions have the same color.

K. Appel and W. Haken proved it in 1977 using a computer.

Mathematicians have been arguing ever since whether or not it's a proof.

Four colors suffice: Proof completed by computer

The Four-Color Theorem: for any map no more than 4 colors are required so that no two adjacent regions have the same color.

K. Appel and W. Haken proved it in 1977 using a computer.

Mathematicians have been arguing ever since whether or not it's a proof.

Four colors suffice: Proof completed by computer

The Four-Color Theorem: for any map no more than 4 colors are required so that no two adjacent regions have the same color.

K. Appel and W. Haken proved it in 1977 using a computer.

Mathematicians have been arguing ever since whether or not it's a proof.

Proof completed with and then verified by computer

Figure: Kepler's Conjecture:
the most efficient way to pack spheres

Figure: Tom Hales giving a lecture
By Slawekb - Own work, CC BY-SA 3.0

Proof published, verified by computer 60 years later

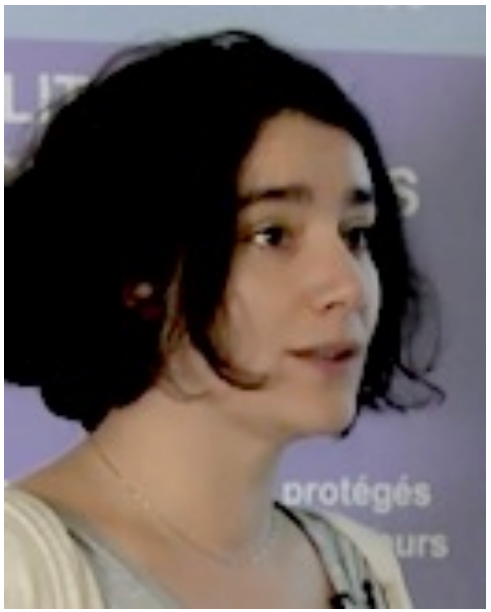


Figure: Assia Mahboubi, one of 15 members of a French-Microsoft collaboration to complete a computer-assisted proof of a famous theorem in group theory

"The Odd Order Theorem... due to W. Feit and J. G. Thompson ... states that every finite group of odd order is solvable.

It is famous ... for the striking contrast between the simplicity of its statement and the unusual length and complexity of its proof.

After a six year collaborative effort, we managed to formalize and machine-check a complete proof of this theorem using the Coq proof assistant."

Proof published, verified by computer 60 years later

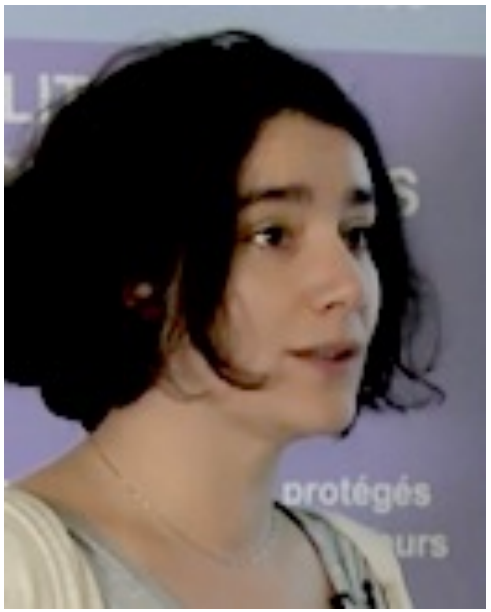


Figure: Assia Mahboubi, one of 15 members of a French-Microsoft collaboration to complete a computer-assisted proof of a famous theorem in group theory

"The Odd Order Theorem... due to W. Feit and J. G. Thompson ... states that every finite group of odd order is solvable.

It is famous ... for the striking contrast between the simplicity of its statement and the unusual length and complexity of its proof.

After a six year collaborative effort, we managed to formalize and machine-check a complete proof of this theorem using the Coq proof assistant."

Proof published, verified by computer 60 years later

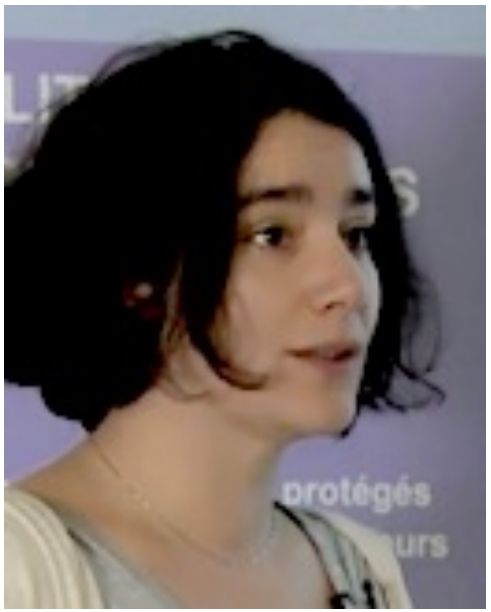


Figure: Assia Mahboubi, one of 15 members of a French-Microsoft collaboration to complete a computer-assisted proof of a famous theorem in group theory

"The Odd Order Theorem... due to W. Feit and J. G. Thompson ... states that every finite group of odd order is solvable.

It is famous ... for the striking contrast between the simplicity of its statement and the unusual length and complexity of its proof.

After a six year collaborative effort, we managed to formalize and machine-check a complete proof of this theorem using the Coq proof assistant."

Proof published, verified by computer 60 years later

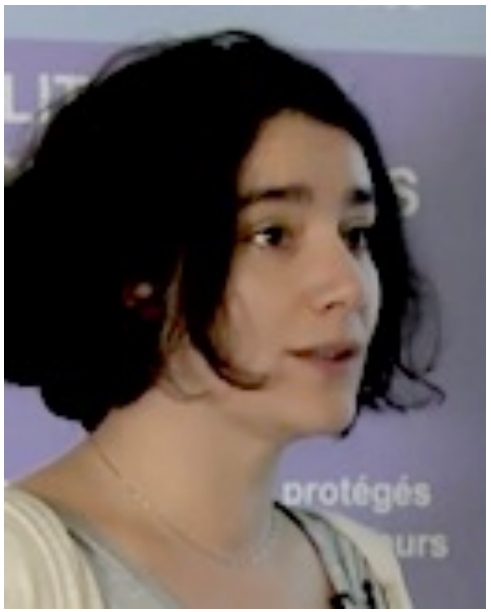


Figure: Assia Mahboubi, one of 15 members of a French-Microsoft collaboration to complete a computer-assisted proof of a famous theorem in group theory

"The Odd Order Theorem... due to W. Feit and J. G. Thompson ... states that every finite group of odd order is solvable.

It is famous ... for the striking contrast between the simplicity of its statement and the unusual length and complexity of its proof.

After a six year collaborative effort, we managed to formalize and machine-check a complete proof of this theorem using the Coq proof assistant."

I see it and I believe it (Plato's *Meno*)

Figure: Bust of Plato, Vatican Museum

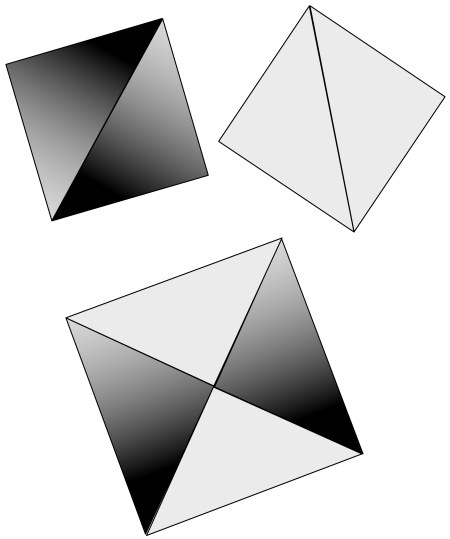
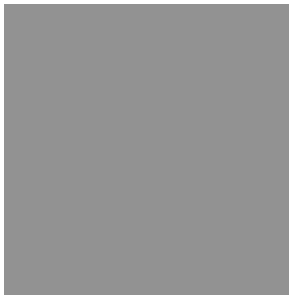


Figure: Dividing a square into two squares

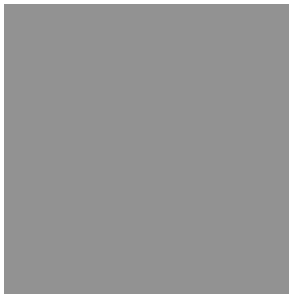
"I see it but I don't believe it": G. Cantor

It looks like there are more points in a square than in a line:



"I see it but I don't believe it": G. Cantor

It looks like there are more points in a square than in a line:

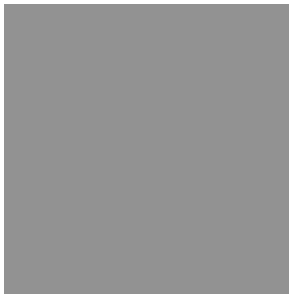


1/1	1/2	1/3	1/4 ...
2/1	2/2	2/3	2/4 ...
3/1	3/2	3/3	3/4 ...
4/1	4/2	4/3	4/4 ....
⋮	⋮	⋮	⋮

1 2 3 4 . . .

"I see it but I don't believe it": G. Cantor

It looks like there are more points in a square than in a line:

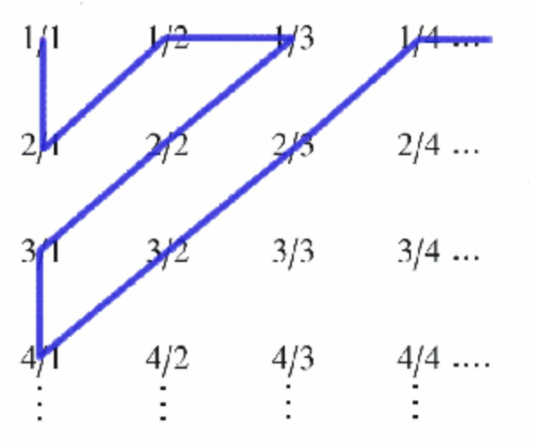


1/1	1/2	1/3	1/4 ...
2/1	2/2	2/3	2/4 ...
3/1	3/2	3/3	3/4 ...
4/1	4/2	4/3	4/4 ....
⋮	⋮	⋮	⋮

1 2 3 4 . . .

"I see it but I don't believe it": G. Cantor

But in fact

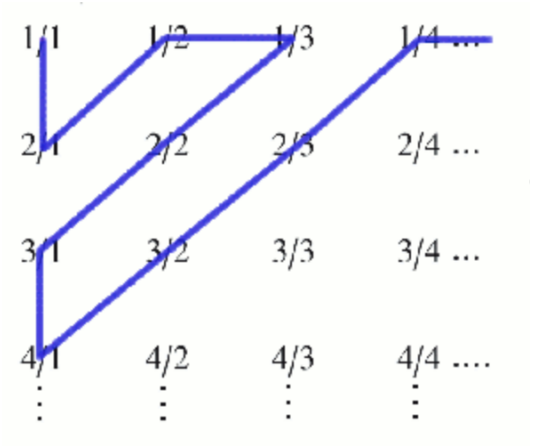


$1 \to 2 \to 3 \to 4 \to \ldots$

Figure: Putting the rational numbers in order

"I see it but I don't believe it": G. Cantor

But in fact



$$1 \to 2 \to 3 \to 4 \to \ldots$$

Figure: Putting the rational numbers in order

(As was explained during the talk, this is obviously NOT Cantor's proof that the interval and the square have the same cardinality, but rather a different theorem of Cantor that is easier to illustrate.)

I don't see it ... but I believe it (.0028% of a formal proof)

(900 K lines of code,
5000+ CPU hours)

```
1094  let h0_lt_gt = prove_by_refinement(
1095    `((y <= #2.01) ==> (y <= &2 * h0)) /\
1096     ((#2.8 <= y) ==> (&2 * h0 <= y)) /\
1097    (( y <= &2) ==> (y <= &2 * h0)) /\
1098      ((sqrt8 <= y) ==> (&2 * h0 <= y)) /\   ((&2 * h0 <= y) ==> (&0 <= y)) /\
1099     ((&2 <= y) ==> (&0 <= y)) /\
1100     ((y <= &2 * hminus) ==> (y <= &2 * h0)) /\
1101     ((&2 * hminus <= y) ==> (&0 <= y))`,
1102    (* {{{ proof *)
1103    [
1104    REWRITE_TAC[Sphere.h0;sqrt8_sqrt2;hm0];
1105      MP_TAC sqrt2_lb;
1106    MP_TAC hminus_gt;
1107    REAL_ARITH_TAC;
1108    ]);;
1109    (* }}} *)
1110
1111  let sqrtxx = prove_by_refinement(
1112    `!x. &0 <= x ==> (sqrt(x * x) = x)`,
1113    (* {{{ proof *)
1114    [
1115      REWRITE_TAC[POW_2_SQRT_ABS;REAL_ARITH `x * x = x pow 2`];
1116    REAL_ARITH_TAC;
1117    ]);;
1118    (* }}} *)
```

Figure: Excerpt,
https://github.com/flyspeck/flyspeck/blob/master/text_formalization/nonlinear/nonlinear_lemma.hl

Can a proof be both cartesian and leibnizian?

Forget for the moment that if we are serious about formalization of proofs and derivation from first principles, then the inference

$$\textit{Understanding} \Rightarrow \textit{Verification}$$

means that hardly any mathematician has ever understood anything....
We are still faced with the unfortunate circumstance that verification does not entail understanding.

(MH, *Do androids prove theorems in their sleep?*)

Can a proof be both cartesian and leibnizian?

Forget for the moment that if we are serious about formalization of proofs and derivation from first principles, then the inference

$$\textit{Understanding} \Rightarrow \textit{Verification}$$

means that hardly any mathematician has ever understood anything.... We are still faced with the unfortunate circumstance that verification does not entail understanding.

(MH, *Do androids prove theorems in their sleep?*)

This is an old question

Massimo Mazzotti, writing about mathematics in Naples in the 1830s:

The "cold algebraists," he says, reduced the once-noble science of mathematics to mere mechanical calculation, without any deeper meaning. While Fergola "sees God behind the circle and the triangle," these atheists "see only the nothingness behind their formulas." ...

Moreover, for the anti-algebraists of Naples, "certainty" was on the side of *intuition*, **not** mechanical calculation.

This is an old question

Massimo Mazzotti, writing about mathematics in Naples in the 1830s:

The "cold algebraists," he says, reduced the once-noble science of mathematics to mere mechanical calculation, without any deeper meaning. While Fergola "sees God behind the circle and the triangle," these atheists "see only the nothingness behind their formulas." ...

Moreover, for the anti-algebraists of Naples, "certainty" was on the side of *intuition*, **not** mechanical calculation.

This is an old question

Massimo Mazzotti, writing about mathematics in Naples in the 1830s:

The "cold algebraists," he says, reduced the once-noble science of mathematics to mere mechanical calculation, without any deeper meaning. While Fergola "sees God behind the circle and the triangle," these atheists "see only the nothingness behind their formulas." ...

Moreover, for the anti-algebraists of Naples, "certainty" was on the side of *intuition*, **not** mechanical calculation.

19th century French suspicion of leibnizian proofs

(Lorraine Daston, writing about mathematics in Paris in the early 19th century)

Analysis functioned as a shortcut; a sort of machinery of symbols that one need only "to arrange on paper, following certain very simple rules, in order to arrive infallibly at new truths." The very efficiency of the mechanical process, which obscured the route by which it achieved its results, rendered it suspect to mathematicians who believed that valid reasoning demanded clear and distinct ideas. ... The metaphor of the blind machine of analysis, which cranks out its results magically and mysteriously, recurs throughout the writings of [French] synthetic geometers.

19th century French suspicion of leibnizian proofs

(Lorraine Daston, writing about mathematics in Paris in the early 19th century)

Analysis functioned as a shortcut; a sort of machinery of symbols that one need only "to arrange on paper, following certain very simple rules, in order to arrive infallibly at new truths." The very efficiency of the mechanical process, which obscured the route by which it achieved its results, rendered it suspect to mathematicians who believed that valid reasoning demanded clear and distinct ideas. ... The metaphor of the blind machine of analysis, which cranks out its results magically and mysteriously, recurs throughout the writings of [French] synthetic geometers.

19th century French suspicion of leibnizian proofs

(Lorraine Daston, writing about mathematics in Paris in the early 19th century)

Analysis functioned as a shortcut; a sort of machinery of symbols that one need only "to arrange on paper, following certain very simple rules, in order to arrive infallibly at new truths." The very efficiency of the mechanical process, which obscured the route by which it achieved its results, rendered it suspect to mathematicians who believed that valid reasoning demanded clear and distinct ideas. ... The metaphor of the blind machine of analysis, which cranks out its results magically and mysteriously, recurs throughout the writings of [French] synthetic geometers.

Mathematics is unambiguous

Math is one of the few institutions we have left free of doublespeak or embellishment or biased opinion. Its words are supposed to mean exactly what they say. Let's keep them that way.

(Manil Suri, *NY Times*, March 5, 2019)

But what exactly does a proof say?

Mathematics is unambiguous

Math is one of the few institutions we have left free of doublespeak or embellishment or biased opinion. Its words are supposed to mean exactly what they say. Let's keep them that way.

(Manil Suri, *NY Times*, March 5, 2019)

But what exactly does a proof say?

Is anything unambiguous?

It is a truth universally acknowledged, that a single man in possession of a good fortune, must be in want of a wife.

(Jane Austen, *Pride and Prejudice*)

It is a universally recognized fact that a man who is lucky should have a wife.

(Google translate, three times via Arabic.)

This is a generally accepted fact. A lucky man must lack a wife.

(Google translate, twice via Chinese)

Is anything unambiguous?

It is a truth universally acknowledged, that a single man in possession of a good fortune, must be in want of a wife.

(Jane Austen, *Pride and Prejudice*)

It is a universally recognized fact that a man who is lucky should have a wife.

(Google translate, three times via Arabic.)

This is a generally accepted fact. A lucky man must lack a wife.

(Google translate, twice via Chinese)

Is anything unambiguous?

It is a truth universally acknowledged, that a single man in possession of a good fortune, must be in want of a wife.

(Jane Austen, *Pride and Prejudice*)

It is a universally recognized fact that a man who is lucky should have a wife.

(Google translate, three times via Arabic.)

This is a generally accepted fact. A lucky man must lack a wife.

(Google translate, twice via Chinese)

What's wrong with human mathematicians?

1. They are sentimental.

Is a proof no more than its verification of the truth of a proposition, which can therefore be translated unambiguously?

Or does a human proof contain more than a logical deduction, just as Austen's sentence is more than its literal meaning?

What's wrong with human mathematicians?

1. They are sentimental.

Is a proof no more than its verification of the truth of a proposition, which can therefore be translated unambiguously?

Or does a human proof contain more than a logical deduction, just as Austen's sentence is more than its literal meaning?

What's wrong with human mathematicians?

1. They are sentimental.

Is a proof no more than its verification of the truth of a proposition, which can therefore be translated unambiguously?

Or does a human proof contain more than a logical deduction, just as Austen's sentence is more than its literal meaning?

What's wrong with human mathematicians?

1. They are sentimental.

Is a proof no more than its verification of the truth of a proposition, which can therefore be translated unambiguously?

Or does a human proof contain more than a logical deduction, just as Austen's sentence is more than its literal meaning?

2. They are mortal (and their minds are not open to inspection).

The Langlands–Tunnell work is sufficiently important that there is little doubt that the experts follow the proof. My question is – "is the experiment reproducible"? Is it science? If there's a nuclear war tomorrow and then one day in the future the paper mathematical literature on our planet is discovered and translated, would the finders be able to put together a full proof of Langlands–Tunnell? Or are there some arguments which are merely "known to the experts"?

Figure: Kevin Buzzard, Professor at Imperial College, London

3. They get bored.

"[Voevodsky's new system] hastens the day when our mathematical literature has been verified mechanically and referees are relieved of the tedium of checking the proofs in articles submitted for publication. "

(Dan Grayson, 2017)

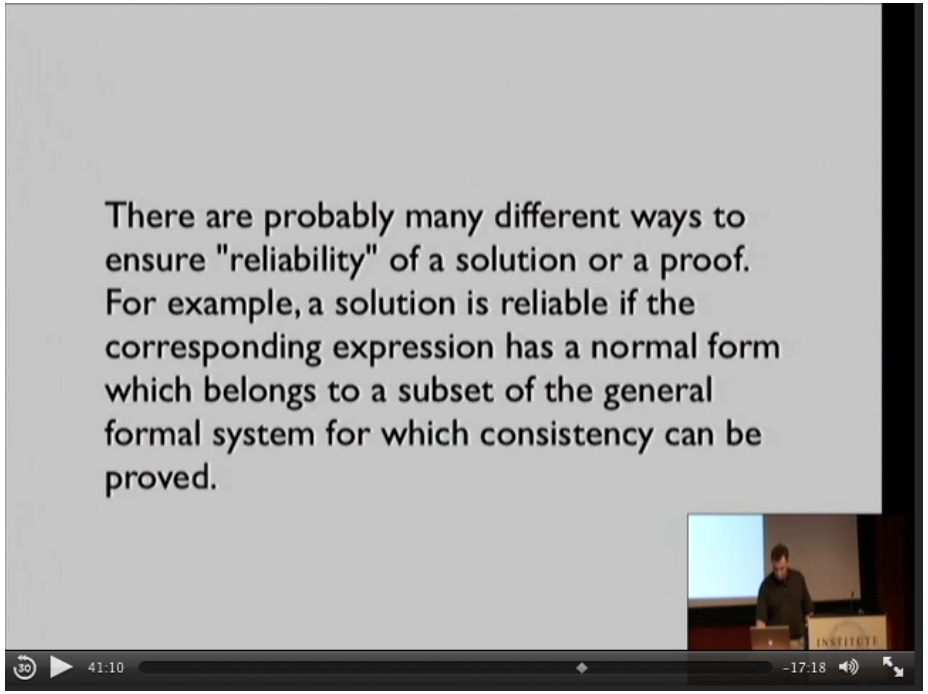


Figure: Vladimir Voevodsky

3. They get bored.

"[Voevodsky's new system] hastens the day when our mathematical literature has been verified mechanically and referees are relieved of the tedium of checking the proofs in articles submitted for publication. "

(Dan Grayson, 2017)

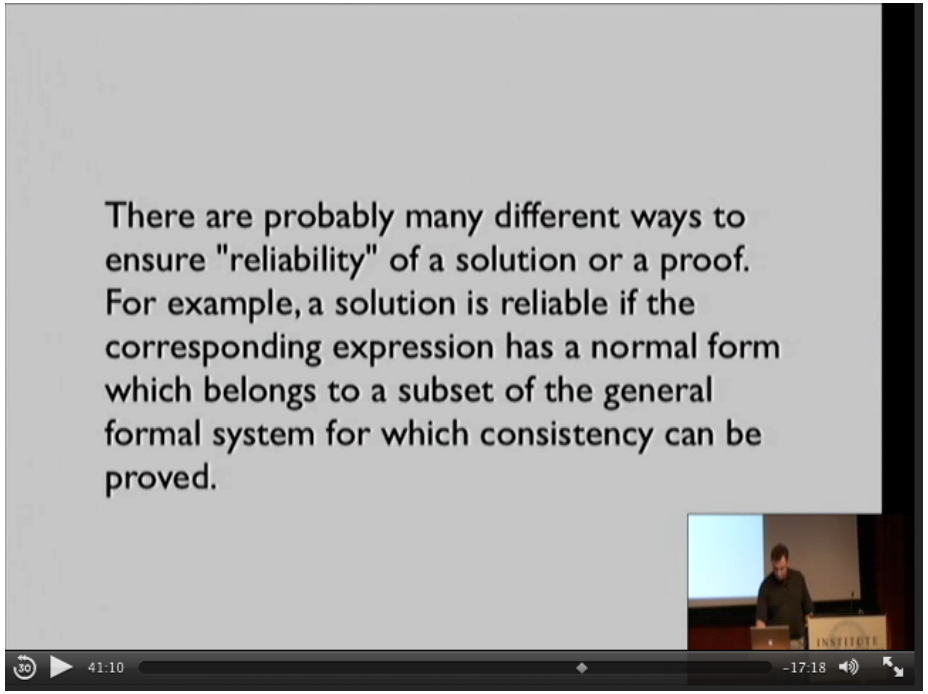
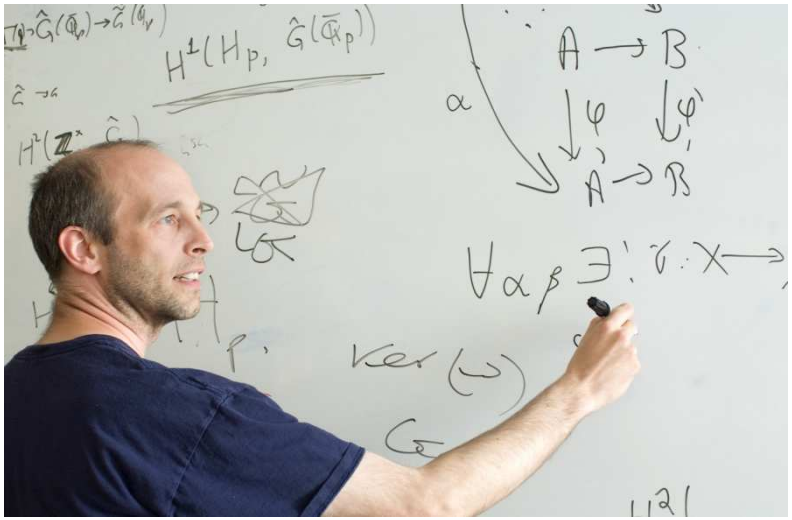


Figure: Vladimir Voevodsky

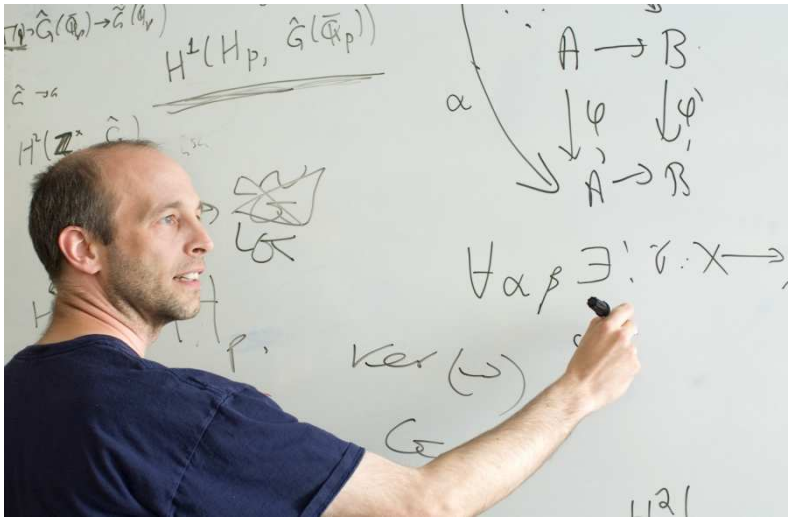
4. They are imprecise.



"OK, now note that the polynomial X has degree 1."

(Kevin is feeding a proof to a computer one line at a time, and the computer checks that each line follows from the preceding line and its built-in library of established results.)

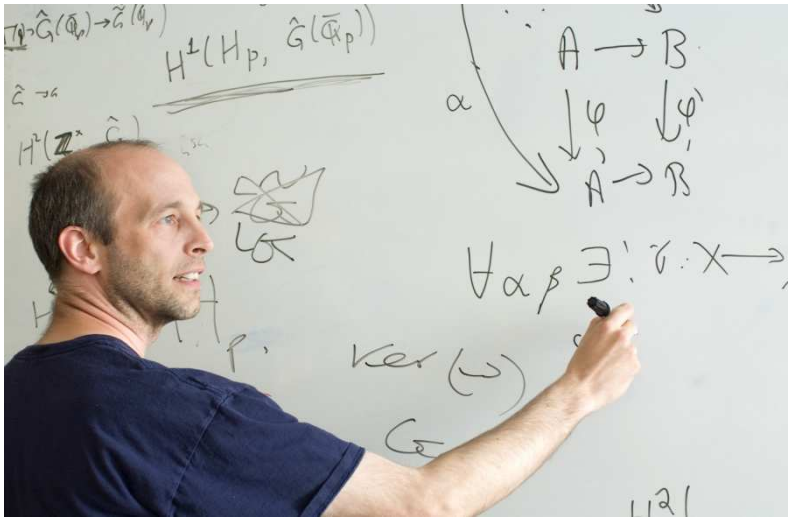
4. They are imprecise.



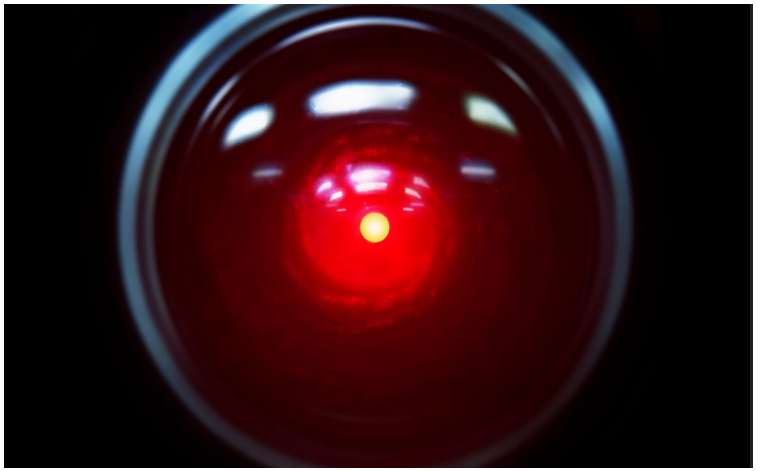
"OK, now note that the polynomial X has degree 1."

(Kevin is feeding a proof to a computer one line at a time, and the computer checks that each line follows from the preceding line and its built-in library of established results.)

4. They are imprecise.

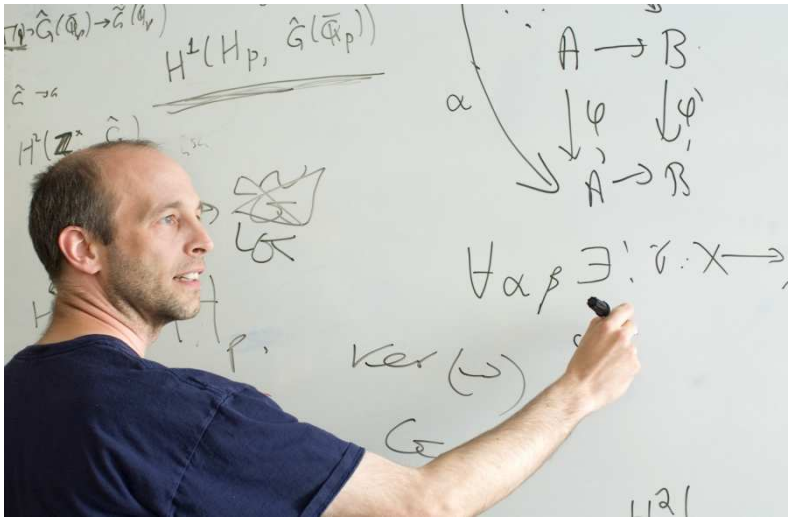


"OK, now note that the polynomial X has degree 1."

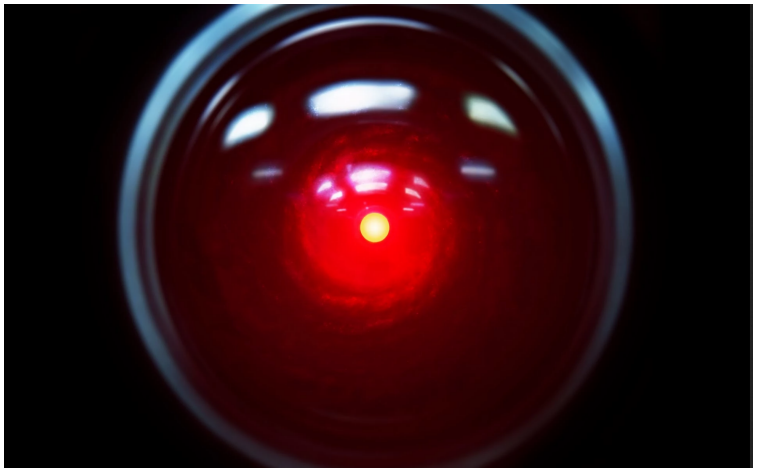


"I'm sorry Kevin, I'm afraid I don't see that."

Who is right: Kevin or HAL?



"X has degree 1, X^2 has degree 2, X^3 has degree 3 . . . "



"I'm sorry Kevin, I'm afraid I don't see that."

Post-humanism

1. Humans are sentimental
2. Humans are mortal
3. Humans get bored
4. Humans are imprecise.

Sounds like an argument for dispensing with humans altogether!

Post-humanism

1. Humans are sentimental
2. Humans are mortal
3. Humans get bored
4. Humans are imprecise.

Sounds like an argument for dispensing with humans altogether!

Post-humanism

1. Humans are sentimental
2. Humans are mortal
3. Humans get bored
4. Humans are imprecise.

Sounds like an argument for dispensing with humans altogether!

Post-humanism

1. Humans are sentimental
2. Humans are mortal
3. Humans get bored
4. Humans are imprecise.

Sounds like an argument for dispensing with humans altogether!

Post-humanism

1. Humans are sentimental
2. Humans are mortal
3. Humans get bored
4. Humans are imprecise.

Sounds like an argument for dispensing with humans altogether!

The temptations of artificial mathematicians

Figure: MIT Professor Sherry Turkle

From its very beginnings, artificial intelligence has worked in this space between a mechanical view of people and a psychological, even spiritual, view of machines.

Norbert Weiner... dreamed in the 1960s that it was "conceptually possible for a human being to be sent over a telegraph line," ... in the mid-1980s, one MIT student mused that ... AI pioneer Marvin Minsky ... wanted to "create a computer beautiful enough that a soul would want to live in it."

The temptations of artificial mathematicians

Figure: MIT Professor Sherry Turkle

From its very beginnings, artificial intelligence has worked in this space between a mechanical view of people and a psychological, even spiritual, view of machines.

Norbert Weiner... dreamed in the 1960s that it was “conceptually possible for a human being to be sent over a telegraph line,” ... in the mid-1980s, one MIT student mused that ... AI pioneer Marvin Minsky ... wanted to “create a computer beautiful enough that a soul would want to live in it.”

How will mechanical mathematicians decide what to prove?

On the basis of algorithms!

"The market itself will be re-imagined as the primary mechanism for the validation of truth. As markets themselves are increasingly turning into algorithmic structures and technologies, the only useful knowledge will be algorithmic."

How will mechanical mathematicians decide what to prove?

On the basis of algorithms!

"The market itself will be re-imagined as the primary mechanism for the validation of truth. As markets themselves are increasingly turning into algorithmic structures and technologies, the only useful knowledge will be algorithmic."

How will mechanical mathematicians decide what to prove?

On the basis of algorithms!

"The market itself will be re-imagined as the primary mechanism for the validation of truth. As markets themselves are increasingly turning into algorithmic structures and technologies, the only useful knowledge will be algorithmic."

How will mechanical mathematicians decide what to prove?

Figure: Achille Mbembe, Cameroonian philosopher, pictures the "21st century political landscape" as a Big Data apocalypse.

"The market itself will be re-imagined as the primary mechanism for the validation of truth. As markets themselves are increasingly turning into algorithmic structures and technologies, the only useful knowledge will be algorithmic.

Instead of people with body, history and flesh, statistical inferences will be all that count. ... The new human being will be constituted through and within digital technologies and computational media."

How will mechanical mathematicians decide what to prove?

Figure: Achille Mbembe, Cameroonian philosopher, pictures the "21st century political landscape" as a Big Data apocalypse.

"The market itself will be re-imagined as the primary mechanism for the validation of truth. As markets themselves are increasingly turning into algorithmic structures and technologies, the only useful knowledge will be algorithmic.

Instead of people with body, history and flesh, statistical inferences will be all that count. ... The new human being will be constituted through and within digital technologies and computational media."

Gowers's alternative

From Sir Timothy Gowers, *The two cultures of mathematics*:

(i) The point of solving problems is to understand mathematics better.

(ii) The point of understanding mathematics is to become better able to solve problems.

Gowers's alternative

From Sir Timothy Gowers, *The two cultures of mathematics*:

(i) The point of solving problems is to understand mathematics better.

(ii) The point of understanding mathematics is to become better able to solve problems.

Solving problems

Gowers belongs to culture (ii).

Gowers, “not particularly happy” at the prospect, has also predicted that by 2099, automation will have put human proof seekers out of business. And he is working with computer scientists to design automatic theorem provers that approximate human intuition.

Pure mathematicians, however, tend to prefer (i).

Solving problems

Gowers belongs to culture (ii).

Gowers, “not particularly happy” at the prospect, has also predicted that by 2099, automation will have put human proof seekers out of business. And he is working with computer scientists to design automatic theorem provers that approximate human intuition.

Pure mathematicians, however, tend to prefer (i).

Solving problems

Gowers belongs to culture (ii).

Gowers, “not particularly happy” at the prospect, has also predicted that by 2099, automation will have put human proof seekers out of business. And he is working with computer scientists to design automatic theorem provers that approximate human intuition.

Pure mathematicians, however, tend to prefer (i).

Understanding

I may think that I understand, but the proof is the check that I have understood, that's all. It is the last stage in the operation ...but it isn't the primary thing at all.

(Sir Michael Atiyah)

The product of mathematics is clarity and understanding. Not theorems, by themselves. ... Their real importance is ... in their role in challenging our understanding, presenting challenges that led to mathematical developments that increased our understanding.

(William Thurston)

Cartesian understanding

"After some hesitation, [Deligne] told [David Ruelle] that what interests him personally are results that he can, by himself and alone, understand in their entirety. This excludes both "theorems obtained with the help of a computer" (like the 4 color problem) and "extremely long ... proofs" (like the classification of finite simple groups).

(At the end of one of his own proofs, published in 1973.)

"Je serais reconnaissant à toute personne ayant compris cette démonstration de me l'expliquer."

Cartesian understanding

"After some hesitation, [Deligne] told [David Ruelle] that what interests him personally are results that he can, by himself and alone, understand in their entirety. This excludes both "theorems obtained with the help of a computer" (like the 4 color problem) and "extremely long ... proofs" (like the classification of finite simple groups).

(At the end of one of his own proofs, published in 1973.)

"Je serais reconnaissant à toute personne ayant compris cette démonstration de me l'expliquer."

The intense experience of understanding

...it happens suddenly: one direction becomes more dense, or more luminous. To experience this intense moment is the reason why I became a mathematician.

(Marie-France Vignéras)

(Can a computer experience an intense moment?)

The intense experience of understanding

...it happens suddenly: one direction becomes more dense, or more luminous. To experience this intense moment is the reason why I became a mathematician.

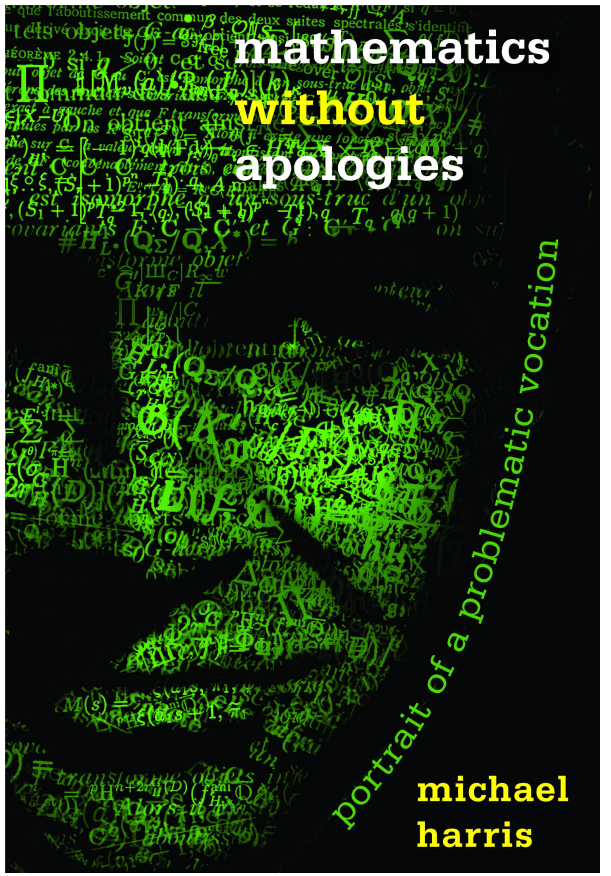
(Marie-France Vignéras)

(Can a computer experience an intense moment?)

Unalienated labor

[W]hy is it a matter of general interest, independently of the uncertain prospects of short- or long-term benefits to human welfare, to have a small group of people working at the limit of their creative powers on something they enjoy? ...

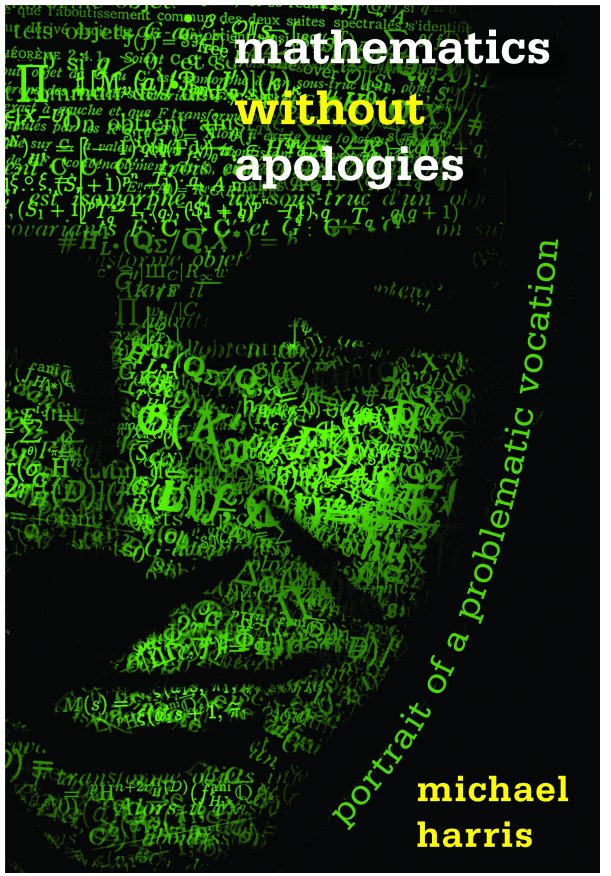
if the notion of "general interest" means anything at all, it should be a matter of general interest that work be a source of pleasure for as many people as possible.



Unalienated labor

[W]hy is it a matter of general interest, independently of the uncertain prospects of short- or long-term benefits to human welfare, to have a small group of people working at the limit of their creative powers on something they enjoy? ...

if the notion of "general interest" means anything at all, it should be a matter of general interest that work be a source of pleasure for as many people as possible.



Unalienated labor = life purpose = way to kill time

Haruzo Hida

"Chinese poems by a decadent Japanese Zen monk of the fifteenth century greatly impressed me; they could be interpreted (with a question mark) to suggest the purpose of one's life could be found only in an enjoyable pastime (or more precisely, a way to kill time), lasting until one's demise. From that time on, I tried in earnest to find such a way to kill time. I finally found one accidentally in the mid-1970s and, after that, became totally addicted to math."

(Haruzo Hida, on receiving the 2019 Steele Prize for Seminal Contribution to Research)

"The one who, in the presence of an arduous problem does not feel his/her enthusiasm grow nor his/her strength increase; the one who, upon approaching the solemn moment of fiat lux so impatiently awaited does not have a soul inundated by that emotion which precedes pleasure, ought to abandon the scientific enterprise, because Nature does not award her favors to the cold-hearted, and frigidity is often an unequivocal sign of impotence ." SRC

Alienated labor is more respectable

...one cannot be surprised when (US) taxpayers object to what they see as wasted money. My own institution [and Hida's], the University of California, used to be "state supported." The state dropped its funding to 11% (2011-2012).

How then can one drum up government (or private!) support for studies in pure mathematics when they are portrayed as having no utility except to give pleasure to a small group of mathematicians?

(Richard Bateman, retired computer scientist, objecting to the review of *Mathematics without Apologies* in the *New York Review of Books*.)

Alienated labor is more respectable

...one cannot be surprised when (US) taxpayers object to what they see as wasted money. My own institution [and Hida's], the University of California, used to be "state supported." The state dropped its funding to 11% (2011-2012).
How then can one drum up government (or private!) support for studies in pure mathematics when they are portrayed as having no utility except to give pleasure to a small group of mathematicians?

(Richard Bateman, retired computer scientist, objecting to the review of *Mathematics without Apologies* in the *New York Review of Books*.)

Is alienated labor more respectable?

How indeed? My advice is to begin by examining what one has been programmed to think and to say, and how this programming – or operant conditioning, the habit of repeating what we hear – diverts us from the path of analysis and inquiry. The word *taxpayer* already conjures up a different conception of society than the word *citizen*. The latter participates in a social contract, the former is an agent of rational choice who views society as an opportunity to choose an investment strategy. I personally don't believe that the "taxpayer" exists as an autonomous political force.

(From the *Mathematics without Apologies* blog, 12/3/2015.)

Alienated labor is more respectable

... books like this, with their self-congratulatory tone, may diminish rather than enhance people's regard for academia in general and mathematics in particular. There are enough people out there with little regard for what university faculty do (witness the fact that a legislator in North Carolina actually proposed a bill that would require faculty in the UNC system, regardless of research expectations, to teach eight classes a year); the last thing we need is to provoke more hostility ...

(From the review of *Mathematics without Apologies* by Mark Hunacek in the *American Mathematical Monthly*.)

Alienated labor is more respectable

... books like this, with their self-congratulatory tone, may diminish rather than enhance people's regard for academia in general and mathematics in particular. There are enough people out there with little regard for what university faculty do (witness the fact that a legislator in North Carolina actually proposed a bill that would require faculty in the UNC system, regardless of research expectations, to teach eight classes a year); the last thing we need is to provoke more hostility ...

(From the review of *Mathematics without Apologies* by Mark Hunacek in the *American Mathematical Monthly*.)

Alienated labor is more respectable

... books like this, with their self-congratulatory tone, may diminish rather than enhance people's regard for academia in general and mathematics in particular. There are enough people out there with little regard for what university faculty do (witness the fact that a legislator in North Carolina actually proposed a bill that would require faculty in the UNC system, regardless of research expectations, to teach eight classes a year); the last thing we need is to provoke more hostility ...

(From the review of *Mathematics without Apologies* by Mark Hunacek in the *American Mathematical Monthly*.)