INTRODUCTION TO HIGHER MATHEMATICS V2000

Homework, week 9, due November 17

1. Basic properties of limits: Dumas-McCarthy, Exercises 5.2, 5.3, 5.6, 5.11.

2. Fibonacci numbers: Dumas-McCarthy, Exercises 5.17, 5.18.

3. Prove from the definitions that

$$\text{(i)} \quad \lim_{x \to 3} 4x - 2 = 10;$$
$$\text{(ii)} \quad \lim_{x \to 2} 4x - 3 \neq 6.$$

4. (a) Let a be an irrational number and fix a positive integer N. Consider a small finite interval (α, β) around a. Let U be the set of rational numbers of the form $\frac{m}{n}$, with $\gcd(m, n) = 1$, $n < N$ and $\alpha \leq \frac{m}{n} \leq \beta$. Show that U is a finite set.

(b) Solve Exercise 5.20 from Dumas-McCarthy.