# HW #4

CALCULUS III

To receive full credit, you must provide a detailed explanation of how you arrived at your answers.

**Question 1.** Find the domain of the vector function and determine whether they are closed, bounded, and/or compact.

(1)
$$\vec{r}(t) = \left\langle e^t, \sqrt{-t}, t^{1/3} \right\rangle$$

(2)
$$\vec{r}(t) = \left\langle \sqrt{t-1}, \sqrt{4-t^2}, \frac{1}{t} \right\rangle$$

**Question 2.** Find the limit, or explain why it does not exist.

(1)
$$\lim_{t \to 0} \left\langle \frac{\sin(t)}{t}, \cos(t), \frac{e^{t^2}-1}{t^2} \right\rangle$$

(2)
$$\lim_{t \to \infty} \left\langle \frac{e^t-1}{t}, \frac{1}{t}, \frac{\sin(t)}{t} \right\rangle$$

**Question 3.** Find a vector function $\vec{r}(t)$ such that the following holds.

$$\vec{r}\,'(t) = \left\langle 2t+1, e^t, -t^2 \right\rangle, \qquad \vec{r}(3) = \langle 10, 0, 10 \rangle$$

**Question 4.** Find a vector equation for the tangent line to the parametric curve

$$x = t^2 - t, \quad y = \frac{e^{t-1}}{t}, \quad z = t\cos(t) - \cos(t)$$

at $(0, 1, 0)$.

**Question 5.** Recall the Mean Value Theorem from Calculus: if $f(t)$ is a differentiable function on $[a, b]$, then there is a number $a \leq c \leq b$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

What about vector functions? Do they satisfy the analogous statement? Namely, for a vector function $\vec{r}(t)$ differentiable on $t$ in $[a, b]$, is there necessarily a number $a \leq c \leq b$ such that

$$\vec{r}\,'(c) = \frac{\vec{r}(b) - \vec{r}(a)}{b - a}?$$

If yes, briefly explain why. If no, exhibit a counterexample.