EXERCISES #20

GLOBAL MAXIMA AND MINIMA, CONTINUED

Exercise 1. Determine whether the following region is closed, bounded or compact.

(1) $\{(x,y) \mid x^2+y^2 \leq 1\}$
(2) $\{(x,y) \mid x^2+y^2 < 1\}$
(3) $\{(x,y) \mid x+y=0\}$
(4) $\{(x,y) \mid x^3+y^3 \leq 1\}$
(5) $\{(x,y) \mid x^4+y^2 \leq 1\}$
(6) $\{(x,y) \mid x^2+y^4+x \leq 1,\ y \geq 0\}$
(7) $\{(x,y,z) \mid x^2+y^2+z \leq 1\}$
(8) $\{(x,y,z) \mid x^2+y^4 \leq z^2\}$
(9) $\{(x,y,z) \mid x^2+y^2+z^2 \leq 2x+2y+2z,\ z \geq 0\}$

Exercise 2. Determine whether f has a global maximum and/or minimum on the region, and if they exist, find the values.

(1) $f(x,y) = xy+x+y$, on $y \geq x^2$
(2) $f(x,y) = x^2+y^2$, on $xy \geq 1$
(3) $f(x,y) = x^2+3y^2-4x-6y$, on $x \geq 0$, $y \geq 0$
(4) $f(x,y) = xye^{-x^2-y^2}$, on $2x-y=0$
(5) $f(x,y) = x^3+y^3-3xy$, on all real numbers x, y
(6) $f(x,y) = 2x^2-2xy+y^2-2x$, on all real numbers x, y
(7) $f(x,y) = x^2+2y$, on $2x+y^2 \leq 3$
(8) $f(x,y) = e^{xy}$, on $x^3+y^3=16$
(9) $f(x,y,z) = xyz$, on $xy+2yz+2zx=12$, $x,y,z \geq 0$
(10) $f(x,y,z) = 4x+2y+z$, on $x^2+y+z^2=1$
(11) $f(x,y,z) = x\ln(x)+y\ln(y)+z\ln(z)-\frac{x+y+z}{3}\ln(xyz)$, on $x,y,z \geq 1$

Exercise 3. Determine whether there is a global maximum or a global minimum of a function f on a region D, and if they exist, find the values.

(1) $f(x,y,z) = z$ on $D = \{(x,y,z) \mid x^2+y^2=z^2,\ x+y+z=24\}$
(2) $f(x,y,z) = x+y+z$ on $D = \{(x,y,z) \mid x^2+z^2 \leq 2,\ x+y \leq 1\}$
(3) $f(x,y,z) = x^2+y^2+z^2$ on $D = \{(x,y,z) \mid x-y=1,\ y^2-z^2=1\}$
(4) $f(x,y,z) = yz+xy$ on $D = \{(x,y,z) \mid xy=1,\ y^2+z^2 \leq 1\}$
(5) $f(x,y,z) = x^2+2y^2+3z^2$ on $D = \{(x,y,z) \mid x+y+z=1,\ x-y+2z=2\}$
(6) $f(x,y,z) = x^2+y^2+z^2$ on $D = \{(x,y,z) \mid 2x+y+2z=9,\ 5x+5y+7z=29\}$
(7) $f(x,y,z) = x^2+y^2+z^2$ on $D = \{(x,y,z) \mid z^2=x^2+y^2,\ x+y-z+1=0\}$
(8) $f(x,y,z) = xyz$ on $D = \{(x,y,z) \mid x+y+z=1,\ x+y-z=0\}$

Exercise 4. Find the distance between two objects.

(1) The distance between the surface $xy^2z^3=2$ and the origin.
(2) The distance between the surface $z=x^2+y^2$ and the point $(1,1,0)$.