

EXERCISES #18

LAGRANGE MULTIPLIERS

Exercise 1. Find the global maximum and minimum values of f subject to the given constraint.

- (1) $f(x, y) = x^2 - y^2$, on $x^2 + y^2 = 1$
- (2) $f(x, y) = xe^y$, on $x^2 + y^2 = 2$
- (3) $f(x, y) = xye^{-x^2-y^2}$, on $x^2 + y^2 = 1$
- (4) $f(x, y, z) = xy^2z$, on $x^2 + y^2 + z^2 = 4$
- (5) $f(x, y, z) = x^2 + y^2 + z^2$, on $x^2 + y^2 + z^2 + xy = 12$
- (6) $f(x, y, z) = x^4 + y^4 + z^4$, on $x^2 + y^2 + z^2 = 1$

Exercise 2. Find the global maximum and minimum values of f on the given region.

- (1) $f(x, y) = x^2 + y^2 + 4x - 4y$, on $x^2 + y^2 \leq 9$
- (2) $f(x, y) = 2x^2 + 3y^2 - 4x - 5$, on $x^2 + y^2 \leq 16$
- (3) $f(x, y) = \sin(x + y)$, on $x^2 + xy + y^2 \leq 3$
- (4) $f(x, y, z) = xyz$, on $x^2 + y^2 + z^2 \leq 1$
- (5) $f(x, y, z) = x^2 + y^2 + z^2$, on $x^4 + y^4 + z^4 \leq 1$
- (6) $f(x, y, z) = x^2 + y^2 + z^2$, on $x^2 + y^2 + z^2 + xy - xz - yz \leq 1$

Exercise 3. Find the global maximum and minimum values of f subject to the given constraint.

- (1) $f(x, y) = x^2y$, on $x^2 + y^2 = 1, y \geq 0$.
- (2) $f(x, y) = e^{-x^2-y^2}(x^2 + 2y^2)$, on $x^2 + y^2 = 4, x + y \geq 0$.
- (3) $f(x, y, z) = xyz$, on $x^2 + y^2 + z^2 = 3, z \geq 0$.

Exercise 4. Find the global maximum and minimum values of f on the given region.

- (1) $f(x, y) = x^3 - 12x + y^3 - 12y$ on the region

$$D = \{(x, y) \mid (x + 2)^2 + (y + 2)^2 \leq 13, x \geq -5\}$$

- (2) $f(x, y) = x + y$ on the region

$$D = \{(x, y) \mid 0 \leq x \leq 1, ex^2 \leq y \leq e^x\}$$

- (3) $f(x, y, z) = x^4 + y + z^2$ on the region

$$D = \{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1, x \geq 0, y \geq 0\}$$

- (4) $f(x, y, z) = xz + yz - xy$ on the region

$$D = \{(x, y, z) \mid z^2 \geq x^2 + y^2, (2 - z)^2 \geq x^2 + y^2, 0 \leq z \leq 2\}$$