Speaker: Tonghai Yang

Title: On a conjecture of Gross and Zagier on algebraicity

Abstract: The automorphic Green function $G_s(z_1, z_2)$ for $SL_2(\mathbb{Z})$, also called the resolvent kernel function for Γ, plays an important role in both analytic and algebra number theory, e.g. in the Gross-Zagier formula and Gross-Kohnen-Zagier formula. It is transcendental in nature, even its CM values are transcendental. It is quite interesting to have the following conjectural algebraicity property. For a weakly holomorphic modular form $f(\tau) = \sum_m c_f(m)q^m$ of weight $-2j$ ($j \geq 0$), consider the linear combination

$$G_{1+j,f}(z_1, z_2) = \sum_{m>0} c_f(-m)m^j G_{1+j}^m(z_1, z_2)$$

where $G_s^m(z_1, z_2)$ is the Hecke correspondence of $G_s(z_1, z_2)$ under the Hecke operator T_m on the first (or second) variable. Gross-Zagier conjectured in 1980s that for any two CM points z_i of discriminants d_i

$$(d_1d_2)^{j/2}G_{j+1,f}(z_1, z_2) = \frac{w_{d_1}w_{d_2}}{4} \cdot \log|\alpha|$$

for some algebraic number α, where w_i is the number of units in O_{d_i}. In this talk, I will describe some progress on this conjecture. If time permits, I will also explain how one method to attack this conjecture also produces an analogue of the Gross-Kohnen-Zagier theorem in Kuga varieties.

In the RTG talk, I will explain regularized theta lifting (Borcherds product) and their CM value formula.