

Speaker: Andreas Wieser

Title: Representations of integral quadratic forms

Abstract: A classical question in number theory asks: which integers are represented by a given quadratic form? More generally, one can ask when an integral quadratic form q in m variables can be represented by another integral quadratic form Q in $n > m$ variables - that is, whether there exists an integer matrix T such that $Q \circ T = q$.

Naturally, a necessary condition is that such a representation exists over the real numbers and modulo N for every positive integer N . In the absence of this local obstruction, does a (global) representation of q by Q exist?

In this talk, we present joint work with Manfred Einsiedler, Elon Lindenstrauss, and Amir Mohammadi in which we establish such a local-global principle in codimension $n - m \geq 3$ using homogeneous dynamics. In fact, we derive this principle from a much more broadly applicable effective equidistribution theorem for semisimple adelic periods. We will also discuss work with Wooyeon Kim and Pengyu Yang in dimensions $m = 2$ and $n = 4$ which fall outside the scope of these equidistribution theorems. The methods developed in this setting provide insights into the mixing conjecture of Michel and Venkatesh.