Speaker: Joseph H. Silverman

Title: Isotrivial Markoff-type K3 surfaces and orbits over finite fields

Abstract: We consider K3 surfaces in $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$ given in affine coordinates by equations of the form

$$\mathcal{W}_{a,b,c,d,e} : ax^2y^2z^2 + b(x^2y^2 + x^2z^2 + y^2z^2) + cxyz + d(x^2 + y^2 + z^2) + e = 0.$$

Each of the projections $\pi_1, \pi_2, \pi_3 : \mathcal{W}_{a,b,c,d,e} \to \mathbb{P}^1$ gives $\mathcal{W}_{a,b,c,d,e}$ the structure of a fibration of genus 1 curves, and the fibrations are isomorphic due to the $\mathcal{S}_3$-symmetry of the equation defining the surface. Let $\mathcal{J}_{a,b,c,d,e} \to \mathbb{P}^1$ denote the Jacobian of any one of these fibrations. We will characterize the parameters $[a, b, c, d, e] \in \mathbb{P}^5$ for which $\mathcal{J}_{a,b,c,d,e}$ is isotrivial, but not split, and we will discuss the automorphism orbit structure of $\mathcal{W}_{a,b,c,d,e}(\mathbb{F}_q)$ in general, and for the isotrivial parameters in particular. We note that the isotrivial $\mathcal{W}_{a,b,c,d,e}$ are natural K3 analogues to classical Markoff surfaces $\mathcal{M}$, for which the three projections $\mathcal{M} \to \mathbb{A}^1$ give $\mathcal{M}$ the structure of a $\mathbb{G}_m$-torsor.