

Speaker: A. Raghuram

Title: Arithmetic Applications of Eisenstein Cohomology

Abstract: The Eichler-Shimura isomorphism provides a bridge between the classical theory of modular forms and the cohomology of arithmetic groups. Just like the space of modular forms decomposes into cusp forms and Eisenstein series, the cohomology of a locally symmetric space decomposes into cuspidal cohomology and Eisenstein cohomology. After introducing Eisenstein cohomology with some examples, I will broadly discuss three applications towards the arithmetic of L-functions. (i) The denominators of Eisenstein classes are related to the numerators of L-values—one sees, for example, the classical Ramanujan’s 691-congruence from this point of view; (ii) Langlands’s constant term theorem can be interpreted in terms of maps in Eisenstein cohomology giving lots of new rationality results on the special values of automorphic L-functions; and (iii) Cross ratio theorem: generalizing a new theorem of Shih-Yu Chen. The three applications were/are in various joint works with (i) Qiyao Yu, (ii) Günter Harder, Chandrasheel Bhagwat, Muthukrishnan Krishnamurthy, Pierre Deligne, etc., and (iii) Harald Grobner, Michael Harris, and Jie Lin, respectively. Of the applications the ideas in (i) and (ii) are originally due to Günter Harder and the power of (iii) comes from the insight in Michael Harris’s Bourbaki seminar in 2024 of how one can get new cases of Deligne’s conjecture on the special values of motivic L-functions.