

Speaker: Bjorn Poonen

Title: Random maximal isotropic subspaces and Selmer groups

Abstract: We show that the p -Selmer group of an elliptic curve is naturally the intersection of two maximal isotropic subspaces in an infinite-dimensional locally compact quadratic space over \mathbb{F}_p . By modeling this intersection as the intersection of a random maximal isotropic subspace with a fixed compact open maximal isotropic subspace, we can explain the known phenomena regarding distribution of Selmer ranks, such as the theorems of Heath-Brown, Swinnerton-Dyer, and Kane for 2-Selmer groups in certain families of quadratic twists, and the average size of 2- and 3-Selmer groups as computed by Bhargava and Shankar. The random model is consistent with Delaunay's heuristics for $\text{Sha}(p)$, and predicts that the average rank of elliptic curves is at most $1/2$. This is joint work with Eric Rains.