VESSELIN DIMITROV AND ZIYANG GAO WIN THE 2022 DAVID GOSS PRIZE

The second David Goss Prize in Number Theory was awarded to Vesselin Dimitrov (University of Toronto, Canada) and Ziyang Gao (Leibniz Universität, Germany) at the JNT Biennial conference in Cetraro, Italy on July 20, 2022. The David Goss Prize (10K USD) will be awarded every two years to mathematicians under the age of 35 for outstanding contributions to number theory. The prize is dedicated to the memory of David Goss who was the former editor in chief of the Journal of Number Theory.

VESSELIN DIMITROV has solved several classical problems in number theory and arithmetic geometry with very original and imaginative methods. His work touches on many topics regarding the heights of algebraic points, G-series, and modular forms. In the following, we list some of his most celebrated accomplishments to date.

The first significant achievement of Dimitrov is his proof of the so-called "Schinzel-Zassenhaus conjecture", a best-possible lower bound (in terms of the degree) on the maximum absolute value among the conjugates of an algebraic integer, not a root of unity. This conjecture is strongly related to (and, in fact, a direct consequence of) the well-known long-standing "Lehmer conjecture" on heights of algebraic numbers. This proof of the conjecture is a breakthrough; indeed, the result and the proof method will have applications in several directions.

Still another major work of Vesselin Dimitrov (in collaboration with F. Calegari and Y. Tang) is a proof of the so-called unbounded denominator conjecture of Atkin and Swinnerton-Dyer (1968) in the modular realm. This predicts that a holomorphic (in $\mathcal{H}$) q-series, which is modular for some finite-index subgroup of $\mathrm{SL}_2(\mathbb{Z})$, meromorphic at the cusps, and having rational number coefficients with bounded denominators, is modular for some congruence subgroup. The proof is quite sophisticated and full of different ingredients (for instance G-series), some of which have their origin in irrational statements about celebrated constants, like zeta-values or more general 'periods.'

ZIYANG GAO has made a remarkable contribution to arithmetic geometry with originality and creativity. His research has touched on several central arithmetic topics, including the special points on Shimura varieties, torsion points on abelian varieties, and rational points on curves. In the following, we describe the contents of these results briefly.

The first significant achievement of Ziyang is about the André–Oort conjecture for mixed Shimura varieties. In this work, he has extended the famous results of two schools for Shimura varieties to mixed Shimura varieties: Edixhoven–Klingler–Ullmo–Yafaev and Pila–Tsimerman–Zannier. Significantly, he was able to extend Tsimerman's work on the André–Oort conjecture to mixed Shimura varieties of abelian type. To have done this work as a new Ph.D. is awe-inspiring as he has learned many techniques from these two schools and put them together.

The second major work of Ziyang Gao is about the solution of the Bogomolov conjecture over function fields in characteristic 0. The original conjecture over number fields was proved in 1996. One crucial ingredient used in the proof in the number field case is an equidistribution theorem of small points on complex abelian varieties in Arakelov theory. This theory is available for general abelian varieties over a field of power series but too weak for application to the Bogomolov conjecture, especially when abelian varieties have good reductions. Ziyang Gao and his collaborator Philipp Habegger developed a new tool to solve this problem. They have combined the technique in Pila–Tsimerman–Zannier with Arakelov's theory. In particular, they have proved a new height inequality about small points.

JOINT WORK OF VESSELIN DIMITROV AND ZIYANG GAO Another brilliant contribution of Dimitrov and Gao to arithmetic geometry concerns Mazur's problem on obtaining uniform bounds on a curve's rational points. More precisely, Mazur conjectured that there is a uniform bound $N(g,d)$ for the number of the rational points for curves C of genus g defined over a number field K of degree d. Recently, Gao, Dimitrov, and Habegger partially solved this conjecture: they got a nice bound in terms of Mordell–Weil's rank of the Jacobians. Thus Mazur's conjecture is the consequence of another conjecture: a folklore conjecture about the uniform bound of Mordell–Weil's classes of principally polarized abelian varieties. To get such a bound, Gao, Dimitrov and Habegger combined all techniques from O-minimality, Arakelov theory, and the work of Faltings–Vojta–Bombieri. After Faltings–Vojta–Bombieri, it suffices to bound the number of rational points with small heights. For this, they used a height inequality together with a new extended Ax-Schanuel theorem from Shimura varieties by Mok–Pila–Tsimerman to mixed Shimura varieties.

The members of the 2022 Prize committe were: Dorian Goldfeld (*Columbia University, USA*), Philippe Michel (*École Polytechnic Lausanne, Switzerland*), Dipendra Prasad (*IITB, India*), Emmanuel Ullmo (*IHES, France*), Umberto Zannier (*Scuola Normale, Italy*), Shou-Wu Zhang (*Princeton University, USA*).