

Speaker: Jordan Ellenberg

Title: Expander graphs, gonality, and Galois representations

Abstract: *(joint work with Chris Hall and Emmanuel Kowalski)* We show that 1-parameter families of abelian varieties over a number field K have few fibers over bounded-degree extensions of K whose mod- p Galois representations have unexpectedly small image. When the abelian variety is an elliptic curve, this result reduces to known facts about gonality of modular curves due to Abramovich and Zograf. The truth of the result is not surprising, but the method of proof is unexpected (at least to us). The argument uses in a central way new results on expansion in Cayley graphs of linear groups over finite fields due to Helfgott, Gill, Pyber-Szabo, Breuillard-Green-Tao, Golesefidy-Varju, etc., in combination with analytic results due to Li and Yau. If time permits, we will rephrase the result in terms of the "Bogomolov property" about low-height points in algebraic extensions of global fields, and speculate about some arithmetic analogues. The paper can be found at:

<http://arxiv.org/abs/1008.3675>