Speaker: Alexandru Buium

Title: Differential Calculus with Integers

Abstract: The aim of the talk is to explain how one can develop an arithmetic analogue of differential calculus in which differentiable functions $x = x(t)$ are replaced by integer numbers n and the derivation operator sending x into dx/dt is replaced by the Fermat quotient operator sending n into $(n-n^p)/p$, where p is a prime integer. The Lie-Cartan geometric theory of differential equations (in which solutions are smooth maps of manifolds) is then replaced by a theory of "arithmetic differential equations" (in which solutions are integral points of algebraic varieties). Some applications to Diophantine geometry over number fields will be explained; an example of such an application involves intersections of finite rank subgroups of elliptic curves with special sets of points arising from modular parametrizations.