

Reading seminar: Kuznetsov components, stability, and moduli spaces

Room: Sophie Germain, 1016

Fridays, 10:30–12:00

October, 2019 – January, 2020

The goal of this seminar is to learn the recent advances on the applications of derived category techniques and stability to the study of certain families of Fano manifolds and their associated hyperkähler manifolds.

The first part of the seminar (consisting approximately of four talks) will be a review of derived category theory: in particular, we will cover the basic definitions (exceptional objects, semiorthogonal decompositions, Kuznetsov components) and the main examples (projective spaces, Grassmannians, quadrics, cubic fourfolds, Gushel–Mukai manifolds, Debarre-Voisin manifolds, homogeneous spaces). Basic references for this part are in the work of Kuznetsov and Perry [Kuz15, Kuz14, Kuz16, KP18a, KP18b]. For the basics on derived category theory, we refer to [Huy06].

In the second part (roughly four talks), we will review stability and moduli spaces for Kuznetsov components. We will start by proving a fundamental result by Abramovich and Polishchuk [AP06] on how to base change t-structures, then we will cover the basic of Bridgeland stability [Bri07], including the latest definition from [BLM+19], and Bridgeland deformation theorem. The examples will be on K3 categories: we will review the general theory, by following the original paper [Bri08], and then the existence result for more general K3 categories [BLMS17, PPZ19]. Finally, the last lecture will be on the basics of moduli spaces of complexes, to arrive to the recent foundational paper [AH-LH18].

The last part of the seminar (two/three talks) will be on applications to cubic fourfolds and, if time permits, Gushel–Mukai manifolds, by first reviewing relative stability and moduli spaces from [BLM+19]

Tentative Plan.

Friday, October 18, 2019: Exceptional collections, semiorthogonal decompositions, examples (Beilinson’s theorem, Grassmannians and Lefschetz decompositions, quadrics, projective bundles, blow-ups). Refs: [Kuz14, Fon13, Kuz15]. Introductory talk to the basic definitions, through classical examples of exceptional collections; review also Kuznetsov’s definition of Lefschetz decomposition, and treat Fonarev’s example of Lefschetz decomposition on Grassmannians. Speaker: *Nicolas Perrin*.

Friday, October 25, 2019: Examples of Fano manifolds of Calabi-Yau type: cubics, Gushel-Mukai, Debarre-Voisin, and Iliev-Manivel manifolds. Refs: [BD85, DK18, DV10,

- [IM11](#)]. Introductory talk to basic examples of Fano manifolds of Calabi-Yau type. Speaker: *Frederic Han*.
- Friday, November 8, 2019:** Kuznetsov components: Serre functors and examples. Construct and study semiorthogonal decompositions in the examples from the previous talk; prove Kuznetsov’s Serre functor theorem. Refs: [[Kuz15](#)]. Speaker: *Céline Bondonrini*.
- Wednesday, November 13, 2019:** Examples of derived equivalences of Kuznetsov components with K3s: \mathcal{C}_{14} , \mathcal{C}_8 , and Gushel–Mukai. Refs: [[AL17](#), [Kuz10](#), [KP18a](#), [KP18b](#)]. Speaker: *Vladimiro Benedetti*.
- Friday, November 22, 2019:** Base change for derived categories and t-structures. Refs: [[Kuz11](#), [AP06](#), [BLM+19](#)]. Speaker: *Emanuele Macrì*.
- Friday, December 13, 2019:** Stability conditions: definition and Bridgeland deformation theorem. Refs: [[Bri07](#), [Bay16](#), [BMS16](#), [BLM+19](#)]. Speaker: *Claire Voisin*.
- Wednesday, December 18, 2019:** Stability conditions on K3 categories. Refs: [[Bri08](#), [BLMS17](#), [PPZ19](#)]. Speaker: *Jieao Song*.
- Friday, January 10, 2020:** Applications of Bridgeland stability on K3 surfaces to algebraic geometry: Mukai’s program. Refs: [[ABS14](#), [Fey17](#)]. Speaker: *Enrico Arbarello*.
- Friday, January 17, 2020:** Moduli spaces, after Lieblich, Toda, Alper, Halpern-Leistner, Heinloth. Refs: [[Lie06](#), [Tod08](#), [AH-LH18](#)]. Speaker: .
- Friday, January 24, 2020:** Relative stability and moduli spaces. Refs: [[BLM+19](#)]. Speaker: .
- Friday, January 31, 2020:** Application to cubic fourfolds and Gushel–Mukai manifolds. Refs: [[BLM+19](#), [LPZ18](#), [Per19](#)]. Speaker: *Giulia Saccà*.

REFERENCES

- [AP06] Abramovich, D., Polishchuk, A., Sheaves of t-structures and valuative criteria for stable complexes, *J. Reine Angew. Math.* **590** (2006), 89–130.
- [AL17] Addington, N., Lehn, M., On the symplectic eightfold associated to a Pfaffian cubic fourfold, *J. Reine Angew. Math.* **731** (2017), 129–137.
- [AH-LH18] Alper, J., Halpern-Leistner, D., Heinloth, J., Existence of moduli spaces for algebraic stacks, eprint [arXiv:1812.01128](#).
- [ABS14] Arbarello, E., Bruno, A., Sernesi, E., Mukai’s program for curves on a K3 surface, *Algebr. Geom.* **1** (2014), 532–557.
- [Bay16] Bayer, A., A short proof of the deformation property of Bridgeland stability conditions, eprint [arXiv:1606.02169](#).
- [BLMS17] Bayer, A., Lahoz, M., Macrì, E., Stellari, P., Stability conditions on Kuznetsov components, eprint [arXiv:1703.10839](#).
- [BLM+19] Bayer, A., Lahoz, M., Macrì, E., Nuer, H., Perry, A., Stellari, P., Stability conditions in family, eprint [arXiv:1902.08184](#).

- [BMS16] Bayer, A., Macrì, E., Stellari, P., The space of stability conditions on abelian threefolds, and on some Calabi-Yau threefolds, *Invent. Math.* **206** (2016), 869–933.
- [BD85] Beauville, A., Donagi, R., La variété des droites d’une hypersurface cubique de dimension 4, *C. R. Acad. Sci. Paris Sér. I Math.* **301** (1985), 703–706.
- [Bri07] Bridgeland, T., Stability conditions on triangulated categories. *Ann. of Math. (2)* **166** (2007), 317–345.
- [Bri08] ———, Stability conditions on K3 surfaces, *Duke Math. J.* **141** (2008), 241–291.
- [DK18] Debarre, O., Kuznetsov, A., Gushel-Mukai varieties: classification and birationalities, *Algebr. Geom.* **5** (2018), 15–76.
- [DV10] Debarre, O., Voisin, C., Hyper-Kähler fourfolds and Grassmann geometry, *J. Reine Angew. Math.* **649** (2010), 63–87.
- [Fey17] Feyzbakhsh, S., Mukai’s program (reconstructing a K3 surface from a curve) via wall-crossing, eprint [arXiv:1710.06692](https://arxiv.org/abs/1710.06692).
- [Fon13] Fonarev, A., On minimal Lefschetz decompositions for Grassmannians, *Izv. RAN. Ser. Mat.* **77** (2013), 203–224.
- [Huy06] Huybrechts, D., *Fourier-Mukai transforms in algebraic geometry*, Oxford Mathematical Monographs, Oxford University Press, Oxford, 2006.
- [IM11] Iliev, A., Manivel, L., Fano manifolds of Calabi-Yau Hodge type, *J. Pure Appl. Algebra* **219** (2015), 2225–2244.
- [Kuz10] Kuznetsov, A., Derived categories of cubic fourfolds, in *Cohomological and geometric approaches to rationality problems*, 219–243, Progr. Math. **282**, Birkhäuser, Boston, 2010.
- [Kuz11] ———, Base change for semiorthogonal decompositions, *Compos. Math.* **147** (2011), 852–876.
- [Kuz14] ———, Semiorthogonal decompositions in algebraic geometry, in *Proceedings of the International Congress of Mathematicians - Seoul 2014*, 635–660, Kyung Moon Sa, Seoul, 2014.
- [Kuz15] ———, Calabi-Yau and fractional Calabi-Yau categories, to appear in: *J. Reine Angew. Math.*, eprint [arXiv:1509.07657](https://arxiv.org/abs/1509.07657).
- [Kuz16] ———, Derived categories view on rationality problems, in *Rationality problems in algebraic geometry*, 67–104, Lecture Notes in Math. **2172**, Springer, Cham, 2016.
- [KP18a] Kuznetsov, A., Perry, A., Derived categories of Gushel-Mukai varieties, *Compositio Math.* **154** (2018), 1362–1406.
- [KP18b] ———, Categorical joins, eprint [arXiv:1804.00144](https://arxiv.org/abs/1804.00144).
- [LPZ18] Li, C., Pertusi, L., Zhao, X., Twisted cubics on cubic fourfolds and stability conditions, eprint [arXiv:1802.01134](https://arxiv.org/abs/1802.01134).
- [Lie06] Lieblich, M., Moduli of complexes on a proper morphism, *J. Algebraic Geom.* **15** (2006), 175–206.
- [Per19] Perry, A., preprint.
- [PPZ19] Perry, A., Pertusi, L., Zhao, X., preprint.
- [Tod08] Toda, Y., Moduli stacks and invariants of semistable objects on K3 surfaces, *Adv. Math.* **217** (2008), 2736–2781.