OIS curves, FX and connected graphs

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Sharing Thoughts

Summary

The task of discounting flows under collaterals from a pool of bootstrapped curve, is the same problem as computing a FX given a list of FXs (via different crosses determined on the fly)

Details

Referring to [MFAT], or [1], it can be shown that the discount factor for a flow at time T in currency i while collateral is in currency k is given by $e^{-(r^i - r^k + c^k)T}$ where notations used is the naturally simplified ones from the following: $r^{(i)}(t)$ is the risk-free continuous compounding zero rate for currency i at time t, $c^{(k)}(t)$ be the continuous compounding collateral return rate for at time t.

Let $\langle\substack{i \\ k}\rangle_T = e^{-(r^i - r^k + c^k)T}$ the discount factor for currency i with collateral k at time T. We will omit T when it is understood.

<u>Proposition 1</u>: for any currencies w, x, y, z we have $\langle\substack{w \\ x}\rangle\langle\substack{y \\ z}\rangle = \langle\substack{w \\ z}\rangle\langle\substack{y \\ x}\rangle$
The proof is straight forward from the discount factor expression.

Hence we see that[1]:
Inversion formula: $\langle\substack{w \\ x}\rangle\langle\substack{x \\ w}\rangle = \langle\substack{w \\ w}\rangle\langle\substack{x \\ x}\rangle$

Transitive formula: $\langle\substack{w \\ x}\rangle\langle\substack{x \\ y}\rangle = \langle\substack{w \\ y}\rangle\langle\substack{x \\ x}\rangle$

Consider the task of computing the discount factor $\langle\substack{i \\ k}\rangle$ among a given collection of currencies. Suppose also that for any currency x, $\langle\substack{x \\ x}\rangle$ is known. That is, each domestic collateral return rate is known in its respective domestic market (which seems to be not a harsh assumption). Then, the inversion formula says that if $\langle\substack{w \\ x}\rangle$ is known, $\langle\substack{x \\ w}\rangle$ is also known. The transitive formula says, modulo the term $\langle\substack{x \\ x}\rangle$, the DF behaves like an FX.

Following the tradition of FX, if we put an arrow from x to y whenever $\langle\substack{x \\ y}\rangle$ is given / known, a directed graph is formed. Let this graph be denoted by G

<u>Proposition 2</u>: $\langle\substack{i \\ k}\rangle$ can be computed if and only if i and k belong to the same connected component in G

[1] Formulas named by my colleague Arnaud Lederc

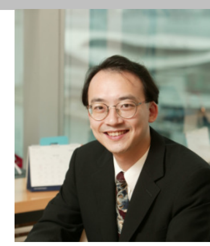
Proof: the “if” part is straight forward from above. Indeed on can trace out the chain of DF involved with any directed path connecting i and k.
The “only if” part can be traced back by the form of the DF expression by omitting the c term. QED

Conclusion: the task of discounting flows under collaterals from a pool of bootstrapped curve, is the same problem as computing a FX given a list of FXs (via different crosses determined on the fly)

Reference

[MFAT] Masaaki Fujii, Akihiko Takahashi, *Choice of collateral currency*, RISK Jan 2011
[1] Tat Sang Fung, *Collateralized Pricing Made Simple*, available http://www.math.columbia.edu/~fts

Contact



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