PRELIMINARY SYLLABUS FOR GU4062 MODERN ANALYSIS II - SPRING 2026

FRANCESCO LIN

Contacts. You can contact me at fl2550@columbia.edu. My office is Mathematics 613.

Hours and location. TuTh 4:10-5:25PM in Mathematics 417.

My office hours. Tu 2-2:30PM and 5:30-6PM, Th 3:30-4PM and 5:30-6PM in my office Mathematics 613. Feel free to stop me for quick questions right after class.

Teaching assistants. Marlet Ma (undergraduate, OH W 4:45-6:45PM UNI: lm3953) and Xinyi Zhang (graduate,OH M 9AM-12PM UNI: xz3272)

Course web Page. All class materials will be available on Coursework; all announcements will be made through the platform.

Prerequisites. Modern Analysis I or equivalent, and proficiency in (finite-dimensional) linear algebra (as in UN2010 Linear algebra) and multivariable differential calculus (as in UN1201 Calculus III).

Contents and goals. This is a continuation of Modern Analysis I. The general goal of the class is to understand the geometric and analytical properties of (infinite dimensional) vector spaces. We will consider concrete examples of spaces of functions, and the emphasis will be placed on *both* theoretical and computational aspects of the theory.

The first three quarters of the class will be devoted to the following topics:

(1) Functional analysis: basics of Hilbert and Banach spaces, bounded operators, the Baire category theorem and its applications.
(2) Fourier analysis: Fourier series and their various convergence properties.
(3) Lebesgue measure and integration theory, Fubini-Tonelli's theorem, L^p spaces.

The major goal of this part of the class is understand the concepts underlying the following fundamental theorem, prove it, and see it in action: *Fourier series determines an isometry of Hilbert spaces between* $L^2(\mathbb{T})$ *and* $\ell^2(\mathbb{Z})$.

In the final quarter of the class we will discuss some aspects of non-linear analysis:

(4) the Banach fixed point theorem and its applications to ordinary differential equations and differentiable maps. Basics concepts of dynamical systems and smooth manifolds.

The final goal of the class is to prove the *Brouwer fixed point theorem* using techniques from differential topology.

Textbook. I will post the handwritten notes I use in class after each lecture. No textbook is required, but we will follow parts of Stein-Shakarchi's series *I: Fourier Analysis*, *III: Real Analysis* and *IV: Functional Analysis*; Baby Rudin and Pugh *Real Mathematical Analysis* are also a good reference for parts of the class. The last part of the course will follow Milnor's *Topology from the differentiable viewpoint.*

Assessment

Homework. There will be homework assignments (roughly) every week. You will find the contents and deadlines in the Assignments tab on the course web page. They will be due (via a file upload) on Thursdays at 11PM.

Solutions must be **handwritten**, and uploaded as a **single, readable PDF**. Documents that do not satisfy this requirement will not be taken into consideration. Your solutions should be clearly marked with problem numbers. Your name (first and last) needs to be written clearly and legibly at the beginning of each set. The graders will be instructed to take away points for anything that causes them undue difficulty in grading your homework, including poor presentation, organization, or handwriting.

The course policy is that **homework delays are not accepted**. However, your two lowest homework scores will not count towards your final grade. Use this for unexpected circumstances such as illnesses when you are unable to do or hand in your assignment on time. Do the problems later, on your own, so that you learn the material.

Homework is the best way to stay up to date with the material. You are **encouraged** to collaborate with your classmates, but you should hand in your own copy of the assignment, with solutions written in your own words. Identical copies are a violation of the expected standard of academic integrity and will be dealt with according to university policy.

Exams. The exams will contain both computational problems and proof-based questions. There will be a 75 minutes in-class Midterm on Thursday March 5th covering part (1) and (2) in the class outline. The final exam date is scheduled by the Registrar (the projected date is Tuesday May 12th, 4:10-7PM), and it will cover the entire course with an emphasis on parts (3) and (4).

There will not be make-up exams. You must plan to take the midterm and final exams at the scheduled time. Besides students with disabilities having prior arrangements with ODS, the only exceptions will be for those with an incapacitating illness, a serious family emergency, or situations of comparable gravity. In both cases you will need a note from your advising dean. Incompletes can be granted only by your advising dean and only in the circumstances mentioned above.

Books, notes, calculators, and other electronic devices will not be allowed (or needed) on quizzes or exams. Anyone guilty of academic dishonesty, such as cheating on an exam or helping someone else to cheat, will automatically fail the course and faces further academic discipline.

Students with disabilities. In order to receive disability-related academic accommodations for this course, students must be registered with their school Disability Services (DS) office.

Exam preparation. I will provide a problem sheet roughly one week before the actual exams; these will be close in spirit to the actual test, and it is recommended for you to work on them on your own in a test-like setup. I will provide hints and sketches of solutions.

Grading. Your final score will be computed via the following weights:

- Homework: 10%
- Midterm: 35%
- Final: 55%