GAUGE THEORY AND TOPOLOGY: AN INTRODUCTION

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Location and time. Università di Pisa, Department of Mathematics, Aula Riunioni. Lectures will take place on Tuesday 11-13 and Wednesday 14-16, starting on November 4th, 12 meetings in total (no class on November 25 and 26).

Contents. We will begin by providing the motivation and geometric intuition for the study of some of the most important gauge theoretic equations (instantons, monopoles, vortices):

- From magnetostatics in $\mathbb{R}^3$ to gauge theory.
- Connections, curvature, and characteristic classes.
- Yang-Mills-Higgs theory, Bogomolnyi bounds, topological solitons.

We will then discuss some of the key tools in geometry and analysis used to study gauge theoretic equations, having in mind as a concrete application Taubes' theorem identifying the moduli space of solutions to the vortex equation on a Riemann surface with its symmetric product:

- Sobolev spaces, elliptic PDEs, Fredholm operators.
- Geometry of Riemann surfaces from a gauge-theoretic viewpoint.
- Moment maps in symplectic geometry and gauge theory.
- Proof of Taubes' theorem.

We will conclude with an introduction to spin geometry and the Seiberg-Witten equations, with the goal of explaining why their solutions are called monopoles and why symmetric products play a central role in contemporary low-dimensional topology.

Materials. I will post handwritten notes for the lectures; the plan is to eventually turn them into an introductory graduate level textbook for self-study, so any feedback is very welcome.

References. Some useful books and papers to look at are:

- Roe - Elliptic operators, asymptotic methods and topology.
- Warner - Foundations of Differentiable Manifolds and Lie Groups.
- Donaldson, Kronheimer - The geometry of four-manifolds.
- Atiyah, Bott - The Yang-Mills equations over a Riemann surface.
- Cannas da Silva - Lectures on symplectic geometry.
- Garcia-Prada - A direct existence proof for the vortex equations over a compact Riemann surface.

For a more thorough discussion of the physics background, one can look at:

- Wald - Advanced classical electrodynamics.
- Manton, Sutcliffe - Topological Solitons.
- Witten - From superconductors and four-manifolds to weak interactions.
- Hamilton - Mathematical gauge theory.

Essential prerequisites. Connections of vector bundles, de Rham cohomology, L^2 spaces and Fourier series, basics of homotopy groups and complex analysis. Some familiarity with classical electrodynamics will be helpful for context.

Assessment. Seminar talk. I will provide a list of potential topics as the course progresses.

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