SYNOPSIS OF PAPERS

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There are 2 main research programs I have come to pursue throughout graduate school, which are somewhat related to each other and which I plan to continue pursuing in the future. Let A be any of the various algebras (or path algebras of monoidal categories) appearing in categorification, for instance Soergel calculus or KLR. The first program is to construct A as coherent sheaves on projective varieties, and to thereby study its semisimplification and its relationships to various Lie algebra actions. My third and fourth preprints in graduate school [2, 1] were on this program. The second program is to study BGG resolutions over A, and their relationships to "nil-Koszulity", namely the property that 'half' of an algebra is Koszul. My first two preprints in graduate school [5, 6] were on this program.

1. Paragraph summaries

Here is a very brief blurb on each of my papers/preprints, which are, in reverse chronological order, [1, 2, 6, 5, 4]. [3] is in preparation. These are not summaries of results, but 'moral summaries', so to speak.

The coherent KLR sheaf [1]. This is my fourth paper in graduate school, joint with Matthew Hase-Liu, in some sense expanding on some of the ideas of [2]. This paper constructs a coherent sheaf $\mathcal{E}$ on a product of projective spaces such that affine sections $\mathcal{E}(U)$ recover the KLR algebra $\mathcal{R}_\alpha$. Perhaps the primary significance of this paper is in its geometric explanation of the $\mathfrak{sl}_2$-action on KLR of Elias-Qi, and the identification of the global sections as the image of the Zuckerman functor, i.e. the maximal finite-dimensional $\mathfrak{sl}_2$-submodule. The key philosophy in the paper is to construct affine cellular algebras appearing in categorification as coherent sheaves on projective spaces by utilizing a faithful action on a dominant projective representation; we anticipate sequels in the vein of this program.

Even and odd minimal categorifications of the nilpotent part of classical and quantum $\mathfrak{sl}(2)$ [2]. This is my third paper in graduate school, joint with Mikhail Khovanov and Alvaro L. Martinez. This paper studies minimal/semisimple categorifications of $U_q^+(\mathfrak{sl}_2)$; one key idea is to obtain a finite-dimensional subalgebra by intersecting a positive and a negative version of nil-Hecke inside some suitable parent space.

BGG resolutions, Koszulity, and stratifications, part II: the Jacobi-Trudi algebra [6]. This is my second paper in graduate school. This paper categorifies the Jacobi-Trudi determinant formula by using KLR algebras and Soergel calculus, together with tools from Koszul theory and an elementary shadow of Ayala-Mazel-Gee-Rozenblyum's stratified noncommutative geometry. Some results of this paper are similar to Bowman-Norton-Simental, but while BNS constructs BGG resolutions by directly writing down the terms and differentials, I construct them by instead following the program of [5]. One interesting feature one gains by doing so is that the BGG differentials can be predicted directly from the comultiplication structure on the Koszul dual of a module; in fact, I show how one can use Koszul duality with respect to the nil-algebra of Soergel calculus to directly obtain the BGG resolution. I also produce a monoidal structure on these "Jacobi-Trudi algebras". This paper gives another example (namely Jacobi-Trudi algebras and cyclotomic Soergel calculi) of nil-Koszul algebras, in addition to nil-Brauer.

BGG resolutions, Koszulity, and stratifications, part I: the nilBrauer algebra [5]. This is my first paper in graduate school. This paper categorifies the change-of-basis formula for the split $\imath$-quantum group of rank 1, $U_q^\imath(\mathfrak{sl}_2)$, by using the nil-Brauer algebras of Brundan-Wang-Webster, together with tools from Koszul theory and an elementary shadow of Ayala-Mazel-Gee-Rozenblyum's stratified noncommutative geometry. This is where I began my program of studying BGG resolutions and nil-Koszul algebras. The significance of this paper is the introduction of tools from Koszul theory to produce BGG resolutions in contexts where guessing the resolution may be difficult, for instance because terms of the resolution may be neither big nor small Vermas but something in between, namely extensions of small Vermas.

On eventually periodic sets as minimal additive complements [4]. This paper lies in the subject of additive combinatorics and was written at the Duluth REU program in 2020, when I was an undergrad. I no longer work in this area.

Categorifications of Hermite polynomials [3, In preparation]**.** This will be my fifth paper in graduate school, though it was worked on between [5] and [6], joint with Mikhail Khovanov and Radmila Sazdanović. The other two authors had constructed an algebra categorifying Hermite polynomials years prior; my main contribution was to produce the BGG resolution by using the same tools as [5].

References

[1] Matthew Hase-Liu and Fan Zhou. The coherent KLR sheaf. *arXiv preprint arXiv:2609.15954*, 2026. https://arxiv.org/abs/2609.15954.

[2] Mikhail Khovanov, Alvaro L. Martinez, and Fan Zhou. Even and odd minimal categorifications of the nilpotent part of classical and quantum sl(2). *arXiv preprint arXiv:2609.04567*, 2026. https://arxiv.org/abs/2609.04567.

[3] Mikhail Khovanov, Radmila Sazdanović, and Fan Zhou. Categorifications of Hermite polynomials. *In preparation*.

[4] Fan Zhou. On eventually periodic sets as minimal additive complements. *The Electronic Journal of Combinatorics*, 30(4):#P4.34, Dec. 2023. https://arxiv.org/abs/2010.12162.

[5] Fan Zhou. BGG resolutions, Koszulity, and stratifications, part I: the nilBrauer algebra. *arXiv preprint arXiv:2402.06890*, 2024. https://arxiv.org/abs/2402.06890.

[6] Fan Zhou. BGG resolutions, Koszulity, and stratifications, part II: the Jacobi-Trudi algebra. *arXiv preprint arXiv:2605.09261*, 2026. https://arxiv.org/abs/2605.09261.

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