Syllabus
Math UN3386 Differential Geometry
Fall 2025

Instructor: Elena Giorgi (elena.giorgi@columbia.edu)

Course Hours and Location: Monday and Wednesday, 1:10pm-2:25pm, in room 520 Mathematics

Graduate student TA: Ivan Zelich (iz2174@columbia.edu)
Office hours: Monday from 1 to 4 pm in Math Help Room 406 .

Overview of the Course: This course initiates the study of differential geometry of curves and surfaces.

Topics of the Course: We will focus mainly on the basic structures of differential geometry of curves and surfaces. A more detailed list of topics covered in the class is the following:

1. Curves
    - Parametrized curves, arc length
    - Local theory of curves
    - Global properties of plane curves
2. Surfaces
    - Regular surfaces
    - First fundamental form
    - The geometry of the Gauss map
    - Gauss-Bonnet theorem
    - Exponential map

Textbooks: *"Differential Geometry of curves and surfaces"* by Manfredo P. Do Carmo.

Structure of the course: We will follow chapters 1, 2, 3, 4 of the textbook by Do Carmo.

In addition to the lectures, there will be weekly problem sets. These will be listed on Coursework and should be uploaded on Gradescope. Late homeworks will not be accepted.

It is very important to do all of the problem sets to the best of your ability, as this is the most effective way to absorb the material. While you are welcome to collaborate with your peers, you must attempt all problems on your own and your submitted solutions must be written out individually.

There will be two midterm exams and one final exam. The midterms will be two seventy-five minute exam scheduled on October 1st and November 5th during normal class hours. The final exam will be administered according to the University exam schedule.

The exams will generally follow the material from the problem sets but may include some additional conceptual problems to test your understanding. The midterms will cover roughly one third each of the course material, while the final exam will cover all the material from the course, with slightly more emphasis on the content covered after the midterms.

Grading scheme: The course grades will be computed as follows: 15% Homework, 25% Midterm exam, 25% Midterm exam, 35% Final exam.

Office Hours

- The instructor will hold weekly office hours, 11:30am-12:30pm on Mondays and Wednesdays, in room 606 Mathematics.
- The TA will hold weekly hours in the Math Help Room in room 406 Mathematics (see the schedule here for their office hours) and you are encouraged to come to their hours with any questions or confusions you may have.

Tentative schedule of lectures: This schedule is tentative and may be modified as the course progresses.

1. September 3: Parametrized curves (1-2); Regular curves, Arc Length (1-3)
2. September 8: Vector product in R3 (1-4); Local theory of curves parametrized by arc length I (1-5)
3. September 10: Local theory of curves parametrized by arc length II (1-5)
4. September 15: Global properties of plane curves: isoperimetric inequality, the four vertex theorem (1-7)
5. September 17: Regular surfaces, Inverse images of regular values I (2-2)
6. September 22: Regular surfaces, Inverse images of regular values II (2-2), Change of parameters (2-3)
7. September 24: Differentiable functions on surfaces (2-3)
8. September 29: [Review practice midterm]
9. October 1: [MIDTERM]
10. October 6: The tangent plane, The differential of a map (2-4)
11. October 8: The first fundamental form, Area (2-5)

12. October 13: Orientation of surfaces (2-6)
13. October 15: The definition of the Gauss map and its fundamental properties I (3-2)
14. October 20: The definition of the Gauss map and its fundamental properties II (3-2)
15. October 22: The Gauss map in local coordinates I (3-3)
16. October 27: The Gauss map in local coordinates II (3-3)
17. October 29: [Review practice midterm]
18. November 5: [MIDTERM]
19. November 10: Isometries, Conformal maps (4-2)
20. November 12: The Gauss theorem and the equations of compatibility (4-3)
21. November 17: Parallel transport, Geodesics I (4-4)
22. November 19: Parallel transport, Geodesics II (4-4)
23. November 24: Parallel transport, Geodesics III (4-4)
24. December 1: The Gauss-Bonnet theorem and its applications (4-5)
25. December 3: The exponential map, Geodesic polar coordinates (4-6)
26. December 8: [Review practice exam]