7. $a_1 = 3$, $a_{n+1} = 2a_n - 1$. Each term is defined in terms of the preceding term.

$a_2 = 2a_1 - 1 = 2(3) - 1 = 5$. $a_3 = 2a_2 - 1 = 2(5) - 1 = 9$. $a_4 = 2a_3 - 1 = 2(9) - 1 = 17$.

$a_5 = 2a_4 - 1 = 2(17) - 1 = 33$. The sequence is $\{3, 5, 9, 17, 33, \ldots\}$.

24. Using the last limit law for sequences and the continuity of the square root function,

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \sqrt{\frac{n+1}{9n+1}} = \sqrt{\lim_{n\to\infty} \frac{n+1}{9n+1}} = \sqrt{\lim_{n\to\infty} \frac{1+1/n}{9+1/n}} = \sqrt{\frac{1}{9}} = \frac{1}{3}.$$ Converges

27. $a_n = \cos(n/2)$. This sequence diverges since the terms don't approach any particular real number as $n \to \infty$. The terms take on values between -1 and 1.

32. $a_n = \dfrac{\ln n}{\ln 2n} = \dfrac{\ln n}{\ln 2 + \ln n} = \dfrac{1}{\frac{\ln 2}{\ln n} + 1} \to \dfrac{1}{0+1} = 1$ as $n \to \infty$. Converges

36. $a_n = \ln(n+1) - \ln n = \ln\left(\dfrac{n+1}{n}\right) = \ln\left(1 + \dfrac{1}{n}\right) \to \ln(1) = 0$ as $n \to \infty$ because $\ln$ is continuous. Converges

46. $0 < |a_n| = \dfrac{3^n}{n!} = \dfrac{3}{1} \cdot \dfrac{3}{2} \cdot \dfrac{3}{3} \cdot \cdots \cdot \dfrac{3}{(n-1)} \cdot \dfrac{3}{n} \le \dfrac{3}{1} \cdot \dfrac{3}{2} \cdot \dfrac{3}{n}$ [for $n > 2$] $= \dfrac{27}{2n} \to 0$ as $n \to \infty$, so by the Squeeze Theorem and Theorem 6, $\{(-3)^n/n!\}$ converges to 0.

61. $a_n = \dfrac{1}{2n+3}$ is decreasing since $a_{n+1} = \dfrac{1}{2(n+1)+3} = \dfrac{1}{2n+5} < \dfrac{1}{2n+3} = a_n$ for each $n \ge 1$. The sequence is bounded since $0 < a_n \le \frac{1}{5}$ for all $n \ge 1$. Note that $a_1 = \frac{1}{5}$.

18. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(\sqrt{2})^n}$ is a geometric series with ratio $r = \dfrac{1}{\sqrt{2}}$. Since $|r| = \dfrac{1}{\sqrt{2}} < 1$, the series converges. Its sum is

$$\frac{1}{1 - 1/\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}-1} = \frac{\sqrt{2}}{\sqrt{2}-1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1} = \sqrt{2}\left(\sqrt{2}+1\right) = 2 + \sqrt{2}.$$

20. $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}} = 3 \sum_{n=1}^{\infty} \left(\frac{e}{3}\right)^n$ is a geometric series with first term $3(e/3) = e$ and ratio $r = \dfrac{e}{3}$. Since $|r| < 1$, the series converges. Its sum is $\dfrac{e}{1 - e/3} = \dfrac{3e}{3-e}$.

22. $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{2n-3}$ diverges by the Test for Divergence since $\displaystyle\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n+1}{2n-3} = \frac{1}{2} \ne 0$.

30. $\displaystyle\sum_{k=1}^{\infty} (\cos 1)^k$ is a geometric series with ratio $r = \cos 1 \approx 0.540302$. It converges because $|r| < 1$. Its sum is

$$\frac{\cos 1}{1 - \cos 1} \approx 1.175343.$$

34. $\sum_{n=1}^{\infty} \frac{e^n}{n^2}$ diverges by the Test for Divergence since $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{e^n}{n^2} = \lim_{x\to\infty} \frac{e^x}{x^2} \overset{\text{H}}{=} \lim_{x\to\infty} \frac{e^x}{2x} \overset{\text{H}}{=} \lim_{x\to\infty} \frac{e^x}{2} = \infty \neq 0$.