2. (a) Since $y = \dfrac{1}{2x-1}$ is defined and continuous on $[1,2]$, $\displaystyle\int_1^2 \frac{1}{2x-1}\,dx$ is proper.

(b) Since $y = \dfrac{1}{2x-1}$ has an infinite discontinuity at $x = \frac{1}{2}$, $\displaystyle\int_0^1 \frac{1}{2x-1}\,dx$ is a Type II improper integral.

(c) Since $\displaystyle\int_{-\infty}^{\infty} \frac{\sin x}{1+x^2}\,dx$ has an infinite interval of integration, it is an improper integral of Type I.

(d) Since $y = \ln(x-1)$ has an infinite discontinuity at $x = 1$, $\int_1^2 \ln(x-1)dx$ is a Type II improper integral.

21. $$\int_1^\infty \frac{\ln x}{x}\,dx = \lim_{t\to\infty}\left[\frac{(\ln x)^2}{2}\right]_1^t \quad \left[\begin{matrix}\text{by substitution with}\\ u = \ln x,\, du = dx/x\end{matrix}\right] \quad = \lim_{t\to\infty}\frac{(\ln t)^2}{2} = \infty. \qquad \text{Divergent}$$

35. $$I = \int_0^3 \frac{dx}{x^2-6x+5} = \int_0^3 \frac{dx}{(x-1)(x-5)} = I_1 + I_2 = \int_0^1 \frac{dx}{(x-1)(x-5)} + \int_1^3 \frac{dx}{(x-1)(x-5)}.$$

Now $\dfrac{1}{(x-1)(x-5)} = \dfrac{A}{x-1} + \dfrac{B}{x-5} \quad \Rightarrow \quad 1 = A(x-5) + B(x-1)$.

Set $x = 5$ to get $1 = 4B$, so $B = \frac{1}{4}$. Set $x = 1$ to get $1 = -4A$, so $A = -\frac{1}{4}$. Thus

$$\begin{aligned} I_1 &= \lim_{t\to 1^-}\int_0^t \left(\frac{-\frac{1}{4}}{x-1} + \frac{\frac{1}{4}}{x-5}\right)dx = \lim_{t\to 1^-}\left[-\frac{1}{4}\ln|x-1| + \frac{1}{4}\ln|x-5|\right]_0^t \\ &= \lim_{t\to 1^-}\left[\left(-\tfrac{1}{4}\ln|t-1| + \tfrac{1}{4}\ln|t-5|\right) - \left(-\tfrac{1}{4}\ln|-1| + \tfrac{1}{4}\ln|-5|\right)\right] \\ &= \infty, \quad \text{since } \lim_{t\to 1^-}\left(-\tfrac{1}{4}\ln|t-1|\right) = \infty. \end{aligned}$$

Since I_1 is divergent, I is divergent.

38. $$\begin{aligned} \int_0^1 \frac{e^{1/x}}{x^3}\,dx &= \lim_{t\to 0^+}\int_t^1 \frac{1}{x}e^{1/x}\cdot\frac{1}{x^2}\,dx = \lim_{t\to 0^+}\int_{1/t}^1 ue^u\,(-du) \qquad \left[\begin{matrix}u = 1/x,\\ du = -dx/x^2\end{matrix}\right] \\ &= \lim_{t\to 0^+}\left[(u-1)e^u\right]_1^{1/t} \quad \left[\begin{matrix}\text{use parts}\\ \text{or Formula 96}\end{matrix}\right] = \lim_{t\to 0^+}\left[\left(\frac{1}{t}-1\right)e^{1/t} - 0\right] \\ &= \lim_{s\to\infty}(s-1)e^s \quad [s = 1/t] \quad = \infty. \qquad \text{Divergent} \end{aligned}$$

40. Integrate by parts with $u = \ln x$, $dv = dx/\sqrt{x} \quad \Rightarrow \quad du = dx/x$, $v = 2\sqrt{x}$.

$$\begin{aligned} \int_0^1 \frac{\ln x}{\sqrt{x}}\,dx &= \lim_{t\to 0^+}\int_t^1 \frac{\ln x}{\sqrt{x}}\,dx = \lim_{t\to 0^+}\left(\left[2\sqrt{x}\ln x\right]_t^1 - 2\int_t^1 \frac{dx}{\sqrt{x}}\right) = \lim_{t\to 0^+}\left(-2\sqrt{t}\ln t - 4\left[\sqrt{x}\right]_t^1\right) \\ &= \lim_{t\to 0^+}\left(-2\sqrt{t}\ln t - 4 + 4\sqrt{t}\right) = -4 \end{aligned}$$

since $\displaystyle\lim_{t\to 0^+}\sqrt{t}\ln t = \lim_{t\to 0^+}\frac{\ln t}{t^{-1/2}} \overset{\text{H}}{=} \lim_{t\to 0^+}\frac{1/t}{-t^{-3/2}/2} = \lim_{t\to 0^+}\left(-2\sqrt{t}\right) = 0.$ $\qquad$ Convergent

50. For $x \geq 1$, $\dfrac{2+e^{-x}}{x} > \dfrac{2}{x}$ [since $e^{-x} > 0$] $> \dfrac{1}{x}$. $\displaystyle\int_1^\infty \frac{1}{x}\,dx$ is divergent by Equation 2 with $p = 1 \leq 1$, so

$\displaystyle\int_1^\infty \frac{2+e^{-x}}{x}\,dx$ is divergent by the Comparison Theorem.

52. For $x \geq 0$, $\arctan x < \dfrac{\pi}{2} < 2$, so $\dfrac{\arctan x}{2+e^x} < \dfrac{2}{2+e^x} < \dfrac{2}{e^x} = 2e^{-x}$. Now

$I = \displaystyle\int_0^\infty 2e^{-x}\,dx = \lim_{t\to\infty}\int_0^t 2e^{-x}\,dx = \lim_{t\to\infty}\left[-2e^{-x}\right]_0^t = \lim_{t\to\infty}\left(-\frac{2}{e^t}+2\right) = 2$, so I is convergent, and by comparison,

$\displaystyle\int_0^\infty \frac{\arctan x}{2+e^x}\,dx$ is convergent.

53. For $0 < x \leq 1$, $\dfrac{\sec^2 x}{x\sqrt{x}} > \dfrac{1}{x^{3/2}}$. Now

$I = \displaystyle\int_0^1 x^{-3/2}\,dx = \lim_{t\to 0^+}\int_t^1 x^{-3/2}\,dx = \lim_{t\to 0^+}\left[-2x^{-1/2}\right]_t^1 = \lim_{t\to 0^+}\left(-2+\frac{2}{\sqrt{t}}\right) = \infty$, so I is divergent, and by

comparison, $\displaystyle\int_0^1 \frac{\sec^2 x}{x\sqrt{x}}$ is divergent.

58. Let $u = \ln x$. Then $du = dx/x \;\Rightarrow\; \displaystyle\int_e^\infty \frac{dx}{x(\ln x)^p} = \int_1^\infty \frac{du}{u^p}$. By Example 4, this converges to $\dfrac{1}{p-1}$ if $p > 1$

and diverges otherwise.

62. Let $k = \dfrac{M}{2RT}$ so that $\overline{v} = \dfrac{4}{\sqrt{\pi}}k^{3/2}\displaystyle\int_0^\infty v^3 e^{-kv^2}\,dv$. Let I denote the integral and use parts to integrate I. Let $\alpha = v^2$,

$d\beta = ve^{-kv^2}\,dv \quad\Rightarrow\quad d\alpha = 2v\,dv$, $\beta = -\dfrac{1}{2k}e^{-kv^2}$:

$$I = \lim_{t\to\infty}\left[-\frac{1}{2k}v^2e^{-kv^2}\right]_0^t + \frac{1}{k}\int_0^\infty ve^{-kv^2}\,dv_0^t = -\frac{1}{2k}\lim_{t\to\infty}\left(t^2e^{-kt^2}\right) + \frac{1}{k}\lim_{t\to\infty}\left[-\frac{1}{2k}e^{-kv^2}\right]$$

$$\overset{\text{H}}{=} -\frac{1}{2k}\cdot 0 - \frac{1}{2k^2}(0-1) = \frac{1}{2k^2}$$

Thus, $\overline{v} = \dfrac{4}{\sqrt{\pi}}k^{3/2}\cdot\dfrac{1}{2k^2} = \dfrac{2}{(k\pi)^{1/2}} = \dfrac{2}{[\pi M/(2RT)]^{1/2}} = \dfrac{2\sqrt{2}\sqrt{RT}}{\sqrt{\pi M}} = \sqrt{\dfrac{8RT}{\pi M}}$.