12. $G(x) = \int_x^1 \cos\sqrt{t}\,dt = -\int_1^x \cos\sqrt{t}\,dt \quad \Rightarrow \quad G'(x) = -\frac{d}{dx}\int_1^x \cos\sqrt{t}\,dt = -\cos\sqrt{x}$

18. Let $u = e^x$. Then $\frac{du}{dx} = e^x$. Also, $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$, so

$$y' = \frac{d}{dx}\int_{e^x}^0 \sin^3 t\,dt = \frac{d}{du}\int_u^0 \sin^3 t\,dt \cdot \frac{du}{dx} = -\frac{d}{du}\int_0^u \sin^3 t\,dt \cdot \frac{du}{dx} = -\sin^3 u \cdot e^x = -e^x \sin^3(e^x).$$

40. $\int_1^2 \frac{4+u^2}{u^3}\,du = \int_1^2 (4u^{-3} + u^{-1})\,du = \left[\frac{4}{-2}u^{-2} + \ln|u|\right]_1^2 = \left[\frac{-2}{u^2} + \ln u\right]_1^2 = (-\frac{1}{2} + \ln 2) - (-2 + \ln 1) = \frac{3}{2} + \ln 2$

12. $\int \left(x^2 + 1 + \frac{1}{x^2+1}\right) dx = \frac{x^3}{3} + x + \tan^{-1} x + C$

18. $\int \frac{\sin 2x}{\sin x}\,dx = \int \frac{2\sin x\,\cos x}{\sin x}\,dx = \int 2\cos x\,dx = 2\sin x + C$

13. Let $u = 5 - 3x$. Then $du = -3\,dx$ and $dx = -\frac{1}{3}\,du$, so

$$\int \frac{dx}{5-3x} = \int \frac{1}{u}\left(-\tfrac{1}{3}\,du\right) = -\tfrac{1}{3}\ln|u| + C = -\tfrac{1}{3}\ln|5-3x| + C.$$

19. Let $u = \ln x$. Then $du = \frac{dx}{x}$, so $\int \frac{(\ln x)^2}{x}\,dx = \int u^2\,du = \frac{1}{3}u^3 + C = \frac{1}{3}(\ln x)^3 + C$.

32. Let $u = e^x + 1$. Then $du = e^x\,dx$, so $\int \frac{e^x}{e^x+1}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln(e^x+1) + C$.

43. Let $u = 1 + x^2$. Then $du = 2x\,dx$, so

$$\int \frac{1+x}{1+x^2}\,dx = \int \frac{1}{1+x^2}\,dx + \int \frac{x}{1+x^2}\,dx = \tan^{-1} x + \int \frac{\frac{1}{2}\,du}{u} = \tan^{-1} x + \tfrac{1}{2}\ln|u| + C$$

$$= \tan^{-1} x + \tfrac{1}{2}\ln\left|1+x^2\right| + C = \tan^{-1} x + \tfrac{1}{2}\ln\left(1+x^2\right) + C \quad [\text{since } 1+x^2 > 0].$$

59. Let $u = 1/x$, so $du = -1/x^2\,dx$. When $x = 1$, $u = 1$; when $x = 2$, $u = \frac{1}{2}$. Thus,

$$\int_1^2 \frac{e^{1/x}}{x^2}\,dx = \int_1^{1/2} e^u\,(-du) = -\left[e^u\right]_1^{1/2} = -(e^{1/2} - e) = e - \sqrt{e}.$$