Practice Midterm 2

1. Determine whether the following integrals are converging or diverging. Justify your answer.

   (a) $\displaystyle\int_1^\infty \frac{\cos^2 x + 1}{\sqrt[3]{x}}\, dx$

   (b) $\displaystyle\int_1^\infty \frac{\ln x}{\sqrt{x}}\, dx$

2. Determine whether the sequence converges or diverges. If it converges, find the limit. Justify your answer.

   (a) $a_n = \dfrac{(n!)^2}{(2n)!}$

   (b) $a_n = \dfrac{(-3)^n}{n^n}$

3. Solve the following problems.

   (a) Compute the partial sums of : $\sum_{n=1}^{\infty} \ln\left(\sqrt{1+\frac{1}{n}}\right)$ and determine whether it converges.

   (b) Find the sum of the series $\sum_{n=1}^{\infty} \frac{2^n}{3^{3n-1}}$

4. Determine whether the series is convergent or divergent. Justify your answer.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\ln n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{(n+1)^7}$

5. Determine whether the series is convergent or divergent. Justify your answer.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n [\ln(\ln n)]^2}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+4}}$