2. $\int \sin^6 x \cos^3 x\,dx = \int \sin^6 x \cos^2 x \cos x\,dx = \int \sin^6 x\,(1-\sin^2 x)\cos x\,dx \overset{s}{=} \int u^6(1-u^2)\,du$
$= \int (u^6 - u^8)\,du = \frac{1}{7}u^7 - \frac{1}{9}u^9 + C = \frac{1}{7}\sin^7 x - \frac{1}{9}\sin^9 x + C$

12. Let $u = x$, $dv = \cos^2 x\,dx \quad \Rightarrow \quad du = dx$, $v = \int \cos^2 x\,dx = \int \frac{1}{2}(1+\cos 2x)\,dx = \frac{1}{2}x + \frac{1}{4}\sin 2x$, so

$\int x\cos^2 x\,dx = x\left(\frac{1}{2}x + \frac{1}{4}\sin 2x\right) - \int\left(\frac{1}{2}x + \frac{1}{4}\sin 2x\right)dx = \frac{1}{2}x^2 + \frac{1}{4}x\sin 2x - \frac{1}{4}x^2 + \frac{1}{8}\cos 2x + C$
$= \frac{1}{4}x^2 + \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + C$

13. $\int_0^{\pi/2} \sin^2 x\cos^2 x\,dx = \int_0^{\pi/2} \frac{1}{4}(4\sin^2 x\cos^2 x)\,dx = \int_0^{\pi/2}\frac{1}{4}(2\sin x\cos x)^2 dx = \frac{1}{4}\int_0^{\pi/2}\sin^2 2x\,dx$
$= \frac{1}{4}\int_0^{\pi/2}\frac{1}{2}(1-\cos 4x)\,dx = \frac{1}{8}\int_0^{\pi/2}(1-\cos 4x)\,dx = \frac{1}{8}\left[x - \frac{1}{4}\sin 4x\right]_0^{\pi/2} = \frac{1}{8}\left(\frac{\pi}{2}\right) = \frac{\pi}{16}$

26. $\int_0^{\pi/4}\sec^4\theta\tan^4\theta\,d\theta = \int_0^{\pi/4}(\tan^2\theta+1)\tan^4\theta\sec^2\theta\,d\theta = \int_0^1(u^2+1)u^4\,du \qquad [u=\tan\theta,\ du=\sec^2\theta\,d\theta]$
$= \int_0^1(u^6+u^4)\,du = \left[\frac{1}{7}u^7 + \frac{1}{5}u^5\right]_0^1 = \frac{1}{7}+\frac{1}{5} = \frac{12}{35}$

30. $\int_0^{\pi/3}\tan^5 x\sec^6 x\,dx = \int_0^{\pi/3}\tan^5 x\sec^4 x\sec^2 x\,dx = \int_0^{\pi/3}\tan^5 x\,(1+\tan^2 x)^2\sec^2 x\,dx$
$= \int_0^{\sqrt{3}} u^5(1+u^2)^2\,du \quad [u=\tan x,\ du=\sec^2 x\,dx] \quad = \int_0^{\sqrt{3}} u^5(1+2u^2+u^4)\,du$
$= \int_0^{\sqrt{3}}(u^5+2u^7+u^9)\,du = \left[\frac{1}{6}u^6 + \frac{1}{4}u^8 + \frac{1}{10}u^{10}\right]_0^{\sqrt{3}} = \frac{27}{6}+\frac{81}{4}+\frac{243}{10} = \frac{981}{20}$

Alternate solution:

$\int_0^{\pi/3}\tan^5 x\sec^6 x\,dx = \int_0^{\pi/3}\tan^4 x\sec^5 x\sec x\tan x\,dx = \int_0^{\pi/3}(\sec^2 x - 1)^2\sec^5 x\sec x\tan x\,dx$
$= \int_1^2 (u^2-1)^2 u^5\,du \qquad [u=\sec x,\ du=\sec x\tan x\,dx]$
$= \int_1^2(u^4-2u^2+1)u^5\,du = \int_1^2(u^9-2u^7+u^5)\,du$
$= \left[\frac{1}{10}u^{10} - \frac{1}{4}u^8 + \frac{1}{6}u^6\right]_1^2 = \left(\frac{512}{5} - 64 + \frac{32}{3}\right) - \left(\frac{1}{10} - \frac{1}{4} + \frac{1}{6}\right) = \frac{981}{20}$

31. $\int\tan^5 x\,dx = \int(\sec^2 x - 1)^2\tan x\,dx = \int\sec^4 x\tan x\,dx - 2\int\sec^2 x\tan x\,dx + \int\tan x\,dx$
$= \int\sec^3 x\sec x\tan x\,dx - 2\int\tan x\sec^2 x\,dx + \int\tan x\,dx$
$= \frac{1}{4}\sec^4 x - \tan^2 x + \ln|\sec x| + C \quad [\text{or } \frac{1}{4}\sec^4 x - \sec^2 x + \ln|\sec x| + C]$

38. $\displaystyle\int_{\pi/4}^{\pi/2}\cot^3 x\,dx = \int_{\pi/4}^{\pi/2}\cot x\,(\csc^2 x - 1)\,dx = \int_{\pi/4}^{\pi/2}\cot x\csc^2 x\,dx - \int_{\pi/4}^{\pi/2}\frac{\cos x}{\sin x}\,dx$
$= \left[-\frac{1}{2}\cot^2 x - \ln|\sin x|\right]_{\pi/4}^{\pi/2} = (0 - \ln 1) - \left[-\frac{1}{2} - \ln\frac{1}{\sqrt{2}}\right] = \frac{1}{2} + \ln\frac{1}{\sqrt{2}} = \frac{1}{2}(1-\ln 2)$

45. $\int\sin 5\theta\sin\theta\,d\theta \overset{2b}{=} \int\frac{1}{2}[\cos(5\theta-\theta) - \cos(5\theta+\theta)]\,d\theta = \frac{1}{2}\int\cos 4\theta\,d\theta - \frac{1}{2}\int\cos 6\theta\,d\theta = \frac{1}{8}\sin 4\theta - \frac{1}{12}\sin 6\theta + C$

47. $\displaystyle\int\frac{1-\tan^2 x}{\sec^2 x}\,dx = \int\left(\cos^2 x - \sin^2 x\right)dx = \int\cos 2x\,dx = \frac{1}{2}\sin 2x + C$

56. (a) Let $u = \cos x$. Then $du = -\sin x\,dx \quad\Rightarrow\quad \int \sin x\,\cos x\,dx = \int u(-du) = -\frac{1}{2}u^2 + C = -\frac{1}{2}\cos^2 x + C_1$.

(b) Let $u = \sin x$. Then $du = \cos x\,dx \quad\Rightarrow\quad \int \sin x\,\cos x\,dx = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}\sin^2 x + C_2$.

(c) $\int \sin x\,\cos x\,dx = \int \frac{1}{2}\sin 2x\,dx = -\frac{1}{4}\cos 2x + C_3$

(d) Let $u = \sin x$, $dv = \cos x\,dx$. Then $du = \cos x\,dx$, $v = \sin x$, so $\int \sin x\,\cos x\,dx = \sin^2 x - \int \sin x\,\cos x\,dx$, by Equation 7.1.2, so $\int \sin x\,\cos x\,dx = \frac{1}{2}\sin^2 x + C_4$.

66. (a) We want to calculate the square root of the average value of $[E(t)]^2 = [155\sin(120\pi t)]^2 = 155^2\sin^2(120\pi t)$. First, we calculate the average value itself, by integrating $[E(t)]^2$ over one cycle (between $t = 0$ and $t = \frac{1}{60}$, since there are 60 cycles per second) and dividing by $\left(\frac{1}{60} - 0\right)$:

$$\begin{aligned}
[E(t)]^2_{\text{ave}} &= \frac{1}{1/60}\int_0^{1/60}[155^2\sin^2(120\pi t)]\,dt = 60\cdot 155^2\int_0^{1/60}\tfrac{1}{2}[1-\cos(240\pi t)]\,dt \\
&= 60\cdot 155^2\left(\tfrac{1}{2}\right)\left[t - \tfrac{1}{240\pi}\sin(240\pi t)\right]_0^{1/60} = 60\cdot 155^2\left(\tfrac{1}{2}\right)\left[\left(\tfrac{1}{60}-0\right)-(0-0)\right] = \tfrac{155^2}{2}
\end{aligned}$$

The RMS value is just the square root of this quantity, which is $\frac{155}{\sqrt{2}} \approx 110$ V.

(b) $220 = \sqrt{[E(t)]^2_{\text{ave}}} \quad\Rightarrow$

$$\begin{aligned}
220^2 = [E(t)]^2_{\text{ave}} &= \frac{1}{1/60}\int_0^{1/60} A^2\sin^2(120\pi t)\,dt = 60A^2\int_0^{1/60}\tfrac{1}{2}[1-\cos(240\pi t)]\,dt \\
&= 30A^2\left[t - \tfrac{1}{240\pi}\sin(240\pi t)\right]_0^{1/60} = 30A^2\left[\left(\tfrac{1}{60}-0\right)-(0-0)\right] = \tfrac{1}{2}A^2
\end{aligned}$$

Thus, $220^2 = \frac{1}{2}A^2 \quad\Rightarrow\quad A = 220\sqrt{2} \approx 311$ V.

67. Just note that the integrand is odd $[f(-x) = -f(x)]$.

Or: If $m \neq n$, calculate

$$\int_{-\pi}^{\pi}\sin mx\,\cos nx\,dx = \int_{-\pi}^{\pi}\tfrac{1}{2}[\sin(m-n)x + \sin(m+n)x]\,dx = \frac{1}{2}\left[-\frac{\cos(m-n)x}{m-n} - \frac{\cos(m+n)x}{m+n}\right]_{-\pi}^{\pi} = 0$$

If $m = n$, then the first term in each set of brackets is zero.

68. $\int_{-\pi}^{\pi}\sin mx\,\sin nx\,dx = \int_{-\pi}^{\pi}\frac{1}{2}[\cos(m-n)x - \cos(m+n)x]\,dx$.

If $m \neq n$, this is equal to $\displaystyle\frac{1}{2}\left[\frac{\sin(m-n)x}{m-n} - \frac{\sin(m+n)x}{m+n}\right]_{-\pi}^{\pi} = 0$.

If $m = n$, we get $\int_{-\pi}^{\pi}\frac{1}{2}[1-\cos(m+n)x]\,dx = \left[\frac{1}{2}x\right]_{-\pi}^{\pi} - \left[\dfrac{\sin(m+n)x}{2(m+n)}\right]_{-\pi}^{\pi} = \pi - 0 = \pi$.

69. $\int_{-\pi}^{\pi} \cos mx\, \cos nx\, dx = \int_{-\pi}^{\pi} \frac{1}{2}[\cos(m-n)x + \cos(m+n)x]\, dx.$

If $m \neq n$, this is equal to $\frac{1}{2}\left[\frac{\sin(m-n)x}{m-n} + \frac{\sin(m+n)x}{m+n}\right]_{-\pi}^{\pi} = 0.$

If $m = n$, we get $\int_{-\pi}^{\pi} \frac{1}{2}[1 + \cos(m+n)x]\, dx = \left[\frac{1}{2}x\right]_{-\pi}^{\pi} + \left[\frac{\sin(m+n)x}{2(m+n)}\right]_{-\pi}^{\pi} = \pi + 0 = \pi.$

70. $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, \sin mx\, dx = \frac{1}{\pi}\int_{-\pi}^{\pi}\left[\left(\sum_{n=1}^{m} a_n \sin nx\right)\sin mx\right] dx = \sum_{n=1}^{m} \frac{a_n}{\pi}\int_{-\pi}^{\pi} \sin mx\, \sin nx\, dx.$ By Exercise 68, every term is zero except the mth one, and that term is $\frac{a_m}{\pi} \cdot \pi = a_m.$