

Section 11.5

1. (a) An alternating series is a series whose terms are alternately positive and negative.

(b) An alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ converges if $0 < b_{n+1} \leq b_n$ for all n and $\lim_{n \rightarrow \infty} b_n = 0$. (This is the Alternating Series Test.)

(c) The error involved in using the partial sum s_n as an approximation to the total sum s is the remainder $R_n = s - s_n$ and the size of the error is smaller than b_{n+1} ; that is, $|R_n| \leq b_{n+1}$. (This is the Alternating Series Estimation Theorem.)

3. $\frac{4}{7} - \frac{4}{8} + \frac{4}{9} - \frac{4}{10} + \frac{4}{11} - \dots = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{4}{n+6}$. Now $b_n = \frac{4}{n+6} > 0$, $\{b_n\}$ is decreasing, and $\lim_{n \rightarrow \infty} b_n = 0$, so the series converges by the Alternating Series Test.

6. $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{\ln(n+4)} = \sum_{n=1}^{\infty} (-1)^{n-1} b_n$. Now $b_n = \frac{1}{\ln(n+4)} > 0$, $\{b_n\}$ is decreasing, and $\lim_{n \rightarrow \infty} b_n = 0$, so the series converges by the Alternating Series Test.

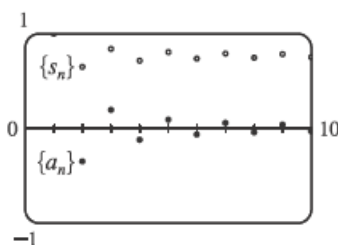
12. $b_n = \frac{e^{1/n}}{n} > 0$ for $n \geq 1$. $\{b_n\}$ is decreasing since $\left(\frac{e^{1/x}}{x}\right)' = \frac{x \cdot e^{1/x}(-1/x^2) - e^{1/x} \cdot 1}{x^2} = \frac{-e^{1/x}(1+x)}{x^3} < 0$ for $x > 0$. Also, $\lim_{n \rightarrow \infty} b_n = 0$ since $\lim_{n \rightarrow \infty} e^{1/n} = 1$. Thus, the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{e^{1/n}}{n}$ converges by the Alternating Series Test.

15. $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^{3/4}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{3/4}}$. $b_n = \frac{1}{n^{3/4}}$ is decreasing and positive and $\lim_{n \rightarrow \infty} \frac{1}{n^{3/4}} = 0$, so the series converges by the Alternating Series Test.

17. $\sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{\pi}{n}\right)$. $b_n = \sin\left(\frac{\pi}{n}\right) > 0$ for $n \geq 2$ and $\sin\left(\frac{\pi}{n}\right) \geq \sin\left(\frac{\pi}{n+1}\right)$, and $\lim_{n \rightarrow \infty} \sin\left(\frac{\pi}{n}\right) = \sin 0 = 0$, so the series converges by the Alternating Series Test.

21.

n	a_n	s_n
1	1	1
2	-0.35355	0.64645
3	0.19245	0.83890
4	-0.125	0.71390
5	0.08944	0.80334
6	-0.06804	0.73530
7	0.05399	0.78929
8	-0.04419	0.74510
9	0.03704	0.78214
10	-0.03162	0.75051



By the Alternating Series Estimation Theorem, the error in the approximation

$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{3/2}} \approx 0.75051$ is $|s - s_{10}| \leq b_{11} = 1/(11)^{3/2} \approx 0.0275$ (to four decimal places, rounded up).

32. If $p > 0$, $\frac{1}{(n+1)^p} \leq \frac{1}{n^p}$ ($\{1/n^p\}$ is decreasing) and $\lim_{n \rightarrow \infty} \frac{1}{n^p} = 0$, so the series converges by the Alternating Series Test.

If $p \leq 0$, $\lim_{n \rightarrow \infty} \frac{(-1)^{n-1}}{n^p}$ does not exist, so the series diverges by the Test for Divergence. Thus, $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p}$

converges $\Leftrightarrow p > 0$.

33. Clearly $b_n = \frac{1}{n+p}$ is decreasing and eventually positive and $\lim_{n \rightarrow \infty} b_n = 0$ for any p . So the series converges (by the Alternating Series Test) for any p for which every b_n is defined, that is, $n+p \neq 0$ for $n \geq 1$, or p is not a negative integer.

34. Let $f(x) = \frac{(\ln x)^p}{x}$. Then $f'(x) = \frac{(\ln x)^{p-1}(p - \ln x)}{x^2} < 0$ if $x > e^p$ so f is eventually decreasing for every p . Clearly

$\lim_{n \rightarrow \infty} \frac{(\ln n)^p}{n} = 0$ if $p \leq 0$, and if $p > 0$ we can apply l'Hospital's Rule $[p+1]$ times to get a limit of 0 as well. So the series

converges for all p (by the Alternating Series Test).

35. $\sum b_{2n} = \sum 1/(2n)^2$ clearly converges (by comparison with the p -series for $p = 2$). So suppose that $\sum (-1)^{n-1} b_n$

converges. Then by Theorem 11.2.8(ii), so does $\sum [(-1)^{n-1} b_n + b_n] = 2(1 + \frac{1}{3} + \frac{1}{5} + \dots) = 2 \sum \frac{1}{2n-1}$. But this

diverges by comparison with the harmonic series, a contradiction. Therefore, $\sum (-1)^{n-1} b_n$ must diverge. The Alternating Series Test does not apply since $\{b_n\}$ is not decreasing.