Name: **Final Exam**

Write your name on this paper. Explain all your answers. Use common sense. Do not use calculators/book/notes. There are nine parts all equally weighted. Here is the first one:

1. Evaluate the following limits.

$$(\mathbf{a})\ \lim_{x\to 0}\frac{x+\sqrt{1+x}-1}{1-\sqrt{1-x}} \qquad (\mathbf{b})\ \lim_{x\to 0}\frac{\sin(x^2)}{x^2} \qquad (\mathbf{c})\ \lim_{x\to\infty}(\ln(x)-\ln(x+1))$$

2. Differentiate the following functions.

$$(\mathbf{a})\ \sqrt{e^x+1} \qquad (\mathbf{b})\ \frac{x^2+1}{x+1} \qquad (\mathbf{c})\ x^{1+\sin(x)}$$

3. Consider the following function

$$f(x) = \begin{cases} 5x, & 0 \leq x \leq 1, \\ x^2 - 6x + 10, & 1 < x \leq 4 \end{cases}$$

(a) Explain why f is continuous on $[0, 4]$.

(b) Find the local and global maxima and minima of f on $[0, 4]$.

4. Graphs.

(a) Find all the horizontal and vertical asymptotes of the function $f(x) = \frac{x+1}{2x+10}$.

(b) What are the intervals where f is concave upwards or downwards and what are the inflection points of the function $f(x) = e^{\frac{2}{3}x^3}$.

5. Theory.

(a) State the fundamental theorem of calculus.

(b) Suppose a continuous function f on $[0, 10]$ has values given by the following table

x	0	1	2	3	4	5	6	7	8	9	10
$f(x)$	1	1	2	2	3	3	4	4	5	5	6

Evaluate the Riemann sum for f with 10 subintervals using as sample points the right endpoints.

6. Examples.

(a) Give an example of a function f on $[0,4]$ with an absolute maximum at 1 and an absolute minimum at 3. No explanation necessary for this part.

(b) Give an example of a function on $[0,1]$ that is not integrable. No explanation necessary for this part.

(c) Give an example of a continuous function f on $[0,10]$ such that $\int_0^{10} f(x)dx = 1/2$.

7. Find the following indefinite integrals.

(a) $\displaystyle\int (x^2 + \sin(2x))dx$ **(b)** $\displaystyle\int \cos(x)e^{\sin(x)}dx$ **(c)** $\displaystyle\int \frac{1+2x}{\sqrt{1-x^2}}dx$

8. Compute the following definite integrals.

(**a**) $\int_1^2 (3x+3)^{1/3} dx$ (**b**) $\int_{-10}^{10} (1+x^{111}) dx$ (**c**) $\int_0^1 x\sqrt{1-x^2} dx$

9. Area and Volume.

(a) Find the area enclosed by the line $y = x + 1$ and the parabola $y = x^2 + 1$.

(b) Find the volume of the solid obtained by rotating the region $x^2 + 5y^2 \leq 1$ around the x-axis. (Hint: Draw a picture of the region, and set $x^2 + 5y^2$ equal to 1 and solve for y.)