LIPSCHITZ LECTURE 4 ACCOMPANYING EXERCISES

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Abstract. Feel free to come by office hours Wednesday and Thursday 3 - 5 p.m. in room 3-040.

(1) We defined a partial order on partitions of size k (i.e., $\{\lambda \vdash k\}$) by

$$\lambda \trianglerighteq \mu \qquad \text{iff} \qquad \lambda_1 + \ldots + \lambda_i \geq \mu_1 + \cdots + \mu_i \quad \forall i.$$

Provide an example of why this is only a partial order. (Hint: k should be at least 6.)

(2) Show that

$$\exp\left\{\sum_{k\geq 1} \frac{p_k(x)p_k(y)}{k}\frac{1-t^k}{1-q^k}\right\} = \sum_{\lambda} \frac{p_\lambda(x)p_\mu(y)}{z_\lambda(q,t)}$$

where $z_\lambda(q,t) = z_\lambda \prod_{i\geq 1} \frac{1-q^{\lambda_i}}{1-t^{\lambda_i}}$ and for $\lambda = 1^{m_1}2^{m_2}\cdots$, $z_\lambda = \prod_{i\geq 1} i^{m_i} m_i!$.