# KNOTS AND PRIMES

## SUMMER 2012 TUTORIAL

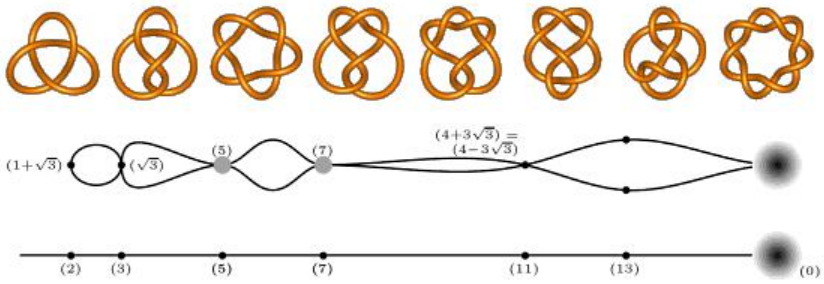
**Dates, time and location.** July 2 to August 10, MWF 7–8 pm in Science Center 109.

**Website.** `http://www.math.harvard.edu/~chaoli/tutorial2012/KnotsAndPrimes.html`

**Instructors.** Chao Li (`chaoli@math.harvard.edu`) and Charmaine Sia (`sia@math.harvard.edu`)

**Tutorial description.** A knot is an embedding of a circle in 3-space. A prime number is a natural number with no positive divisors other than 1 and itself. They are the basic objects of study in knot theory and number theory respectively and have attracted the interest of many mathematicians over the course of history. In knot theory, mathematicians developed various knot invariants to determine when two knots are not equivalent; in number theory, mathematicians use the behavior of various primes to study arithmetic problems.

Surprisingly, these two seemingly unrelated concepts have a deep analogy discovered by Barry Mazur in the 1960s while studying the Alexander polynomial. This initiated the study of what is now known as arithmetic topology. Pursuing deeper connections between knot theory and number theory may raise new points of view and lead to interesting problems and progress in these fields. In this tutorial, we will explore some aspects of this beautiful analogy between knots and primes.




**Tentative syllabus.** We will introduce basic knot theory and number theory, providing many examples and focusing on the intuition and motivation behind the concepts introduced, with the aim of discussing analogies between

- knots and primes,
- 3-manifolds and number rings,
- fundamental groups and Galois groups,
- linking numbers and quadratic reciprocity,
- the Alexander polynomial and Iwasawa theory.

**Reading material.** Reference text: *Knots and Primes: An Introduction to Arithmetic Topology* by Masanori Morishita, Universitext, Springer, 2012. Lecture notes will be provided.

**Possible final paper topics.**

- Analogies between decompositions of knots and primes
- Analogies between homology groups and class groups
- Genus theory of knots and primes
- Arithmetic duality theorems
- Milnor invariants and multiple residue symbols

**Prerequisites.** Basic knowledge of the fundamental group on the level of Math 131 and rings, fields and Galois theory on the level of Math 123.