Midterm I

Problem 1.

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Problem 2.

b, e, c, a, d

Problem 3.

b, e, c, a, d

Problem 4.

(1) $\lim_{x\to 2} \frac{x^2-4}{x^2-5x+6} = \lim_{x\to 2} \frac{x+2}{x-3} = -4.$

(2) $\lim_{x\to 0} \left(\frac{1}{x} - \frac{2}{x^2+2x}\right) = \lim_{x\to 0} \frac{(x+2)-2}{x^2+2x} = \lim_{x\to 0} \frac{1}{x+2} = 1/2.$

(3) $\lim_{x\to 0} \frac{\sqrt{1+2x}-\sqrt{1-2x}}{x} = \lim_{x\to 0} \frac{(1+2x)-(1-2x)}{x(\sqrt{1+2x}+\sqrt{1-2x})} = \lim_{x\to 0} \frac{4}{\sqrt{1+2x}+\sqrt{1-2x}} = 2.$

Problem 5.

(1) $3x^2 - x^{-4/3}$ (power rule)

(2) $\frac{xe^x - 1}{(x+1)^2}$ (quotient rule)

(3) $-8x\sin x + 8\cos x$ (product rule).

Problem 6.

Since $(x^3-x)' = 3x^2-1$, we know the slope of the tangent line of $y = x^3 - x$ at $x = a$ is given by $3a^2-1$. Parallel lines have the same slopes, so we find

$$3a^2 - 1 = 11.$$

Hence $3a^2 = 12$, i.e., $a = \pm 2$. This gives two points $(a, f(a)) = \pm(2,6)$, so there are two such lines by point-slope formula:

$$y = 11(x-2)+6, \quad y = 11(x+2)-6.$$

Problem 7.

Horizontal asymptote: $y = 3$, since dividing x^2 in both the top and bottom we obtain

$$\lim_{x\to\infty} \frac{2+3x^2}{6-8x+x^2} = \lim_{x\to\infty} \frac{2/x^2+3}{6/x^2-8/x+1} = 3.$$

Vertical asymptotes: $x = 2$, $x = 4$, since $f(x) = \frac{2+3x^2}{6-8x+x^2} = \frac{2+3x^2}{(2-x)(4-x)}$. Setting the bottom $(2-x)(4-x) = 0$ gives $x = 2, 4$ and so

$$\lim_{x\to 2} f(x) = \lim_{x\to 4} f(x) = \infty.$$