Assignment 11

Due Monday, May 2, 2011

(1) Prove that any smooth projective plane curve is an algebraic curve.

(2) Let $D = -2p$, where p is a point in $\mathbb{P}^1$, and let $\alpha_D : \mathcal{M}(\mathbb{P}^1) \to \mathcal{T}[D](\mathbb{P}^1)$ be defined in Miranda's book. Let z_p be the local coordinate on $\mathbb{P}^1$ at p, so that $z_p^k \cdot p \in \mathcal{T}[D](\mathbb{P}^1)$ for $k \leq 1$. Prove that $z_p^k \cdot p$ is in the image of α_D iff $k \leq 0$.

(3) Consider a commutative diagram:

$$\begin{array}{ccccccccc}
0 & \longrightarrow & A_1 & \xrightarrow{f_1} & B_1 & \xrightarrow{g_1} & C_1 & \longrightarrow & 0 \\
 & & \alpha\big\downarrow & & \beta\big\downarrow & & \gamma\big\downarrow & & \\
0 & \longrightarrow & A_2 & \xrightarrow{f_2} & B_2 & \xrightarrow{g_2} & C_2 & \longrightarrow & 0
\end{array}$$

where the rows are short exact sequence of abelian groups, and α, β, γ are surjective group homomorphisms. Show that there is a short exact sequence of abelian groups

$$0 \to \mathrm{Ker}(\alpha) \to \mathrm{Ker}(\beta) \to \mathrm{Ker}(\gamma) \to 0.$$

(4) Let ω be a meromorphic 1-form defined on a Riemann surface X. Suppose that $\phi_1 : U_1 \to V_1$ and $\phi_2 \to U_2 \to V_2$ are two complex charts on X, such that $\phi_1(p) = \phi_2(p) = 0$ for some $p \in U_1 \cap U_2$. Suppose that $k = \mathrm{ord}_p(\omega) < 0$, and

$$\begin{aligned}
\omega_{\phi_1} &= \Big(\sum_{n=k}^{\infty} a_n z^n\Big) dz, \\
\omega_{\phi_2} &= \Big(\sum_{n=k}^{\infty} b_n w^n\Big) dw,
\end{aligned}$$

where z is the coordinate on V_1 and w is the coordinate on V_2. Show that $a_{-1} = b_{-1}$.