Math W4045. Algebraic Curves **Spring 2011**

Assignment 10

Due Monday, April 25, 2011

Notation: $\ell(D) = \dim_{\mathbb{C}} L(D)$.

(1) Let D be a divisor of degree 0 on a compact Riemann surface. Show that if $D \not\sim 0$, then $L(D) = \{0\}$.

(2) Suppose that X is a compact Riemann surface and $D > 0$ is a strictly positive divisor on X such that $\ell(D) = 1 + \deg(D)$. Prove that X is isomoprhic to $\mathbb{P}^1$. (Hint: first show that there exists a point $p \in X$ such that $\ell(p) = 2$.)

(3) Let $X = \mathbb{C}/L$ be a complex torus. Let $D, E \in \mathrm{Div}(X)$. Suppose that $\deg D = d > 1$. Show that there exist $p, q \in X$ that $D \sim (d-1)p + q$ and $E(p) = E(q) = 0$.

(4) Let D be a divisor on an algebraic curve of genus g. Suppose that $\deg D = 2g - 2$ and $\ell(D) = g$. Show that D is a canonical divisor. (Hint: use the second form of the Riemann-Roch theorem on page 192 of Miranda's book.)