Math W4045. Algebraic Curves **Spring 2011**

Assignment 3

Due Monday, February 14, 2011

(1) Let $g(z,w), h(z,w) \in \mathbb{C}[z,w]$ be homogenous polynomials of the same degree, where $h(z,w)$ is not the zero polynomial. Define $W = \{[z : w] \in \mathbb{P}^1 \mid h(z,w) \neq 0\}$, which is a nonempty open subset of $\mathbb{P}^1$.

(a) Show that

$$\frac{g}{h} : W \to \mathbb{C}, \quad [z : w] \mapsto \frac{g(z,w)}{h(z,w)}$$

is a holomorphic function on W.

(b) Show that $\frac{g}{h}$ is a meromorphic function on $\mathbb{P}^1$.

(2) Let $X = \{(z,w) \in \mathbb{C}^2 \mid f(z,w) = 0\}$ be a smooth irreducible affine plane curve, where $f(z,w) \in \mathbb{C}[z,w]$ is a nonconstant irreducible polynomial.

(a) Supposet that $g(z,w), h(z,w) \in \mathbb{C}[z,w]$, and that $h(z,w)$ is not divisible by $f(z,w)$. Define $W = \{(z,w) \in X \mid h(z,w) \neq 0\}$, which is a nonempty open subset of X. Show that

$$\frac{g}{h} : W \to \mathbb{C}, \quad (z,w) \mapsto \frac{g(z,w)}{h(z,w)}$$

is a holomorphic function on W. (In particular, when $h = 1$, we conclude that the holomorphic function $g(z,w)$ on $\mathbb{C}^2$ restricts to a holomorphic function on X.)

(b) Show that $\frac{g}{h}$ is a meromorphic function on X.

(3) Let $X = \{[x : y : z] \in \mathbb{P}^2 \mid F(x,y,z) = 0\}$ be a smooth projective plane curve, where $F \in \mathbb{C}[x,y,z]$ is a nonconstant irreducible homogeneous polynomial. Let $G(x,y,z), H(x,y,z) \in \mathbb{C}[x,y,z]$ be homogeneous polynomials of the same degree, where $H(x,y,z)$ is not divisible by $F(x,y,z)$. Define $W = \{[x : y : z] \in X \mid H(x,y,z) \neq 0\}$, which is a nonempty open subset of X.

(a) Show that

$$\frac{G}{H} : W \to \mathbb{C}, \quad [x : y : z] \mapsto \frac{G(x,y,z)}{H(x,y,z)}$$

is a holomorphic function on W.

(b) Show that $\frac{G}{H}$ is a meromorphic function on X.