Math W4045. Algebraic Curves **Spring 2011**

Assignment 2

Due Monday, February 7, 2011

(1) Prove that the quotient topology on $\mathbb{P}^n = (\mathbb{C}^{n+1} - \{0\})/\mathbb{C}^*$ (defined in class) is Hausdorff.

(2) An n-dimensional complex torus is a group of the form $\mathbb{C}^n/L$, where L is a subgroup of $\mathbb{C}^n$ generated by $2n$ vectors which are linearly independent over $\mathbb{R}$. Prove that an n-dimensional complex torus is a compact n-dimensional complex manifold.

(3) For which values of $\lambda \in \mathbb{C}$ are the following projective plane curves smooth? Find the singular points when they exist.

(a) $y^2z = x(x-z)(x-\lambda z)$.

(b) $x^3 + y^3 + z^3 + 3\lambda xyz = 0$.

(4) Show that the line in $\mathbb{P}^2$ through the points $[0 : 1 : 1]$ and $[t : 0 : 1]$ meets the projective plane curve X defined by

$$x^2 + y^2 - z^2 = 0$$

in the two points $[0 : 1 : 1]$ and $[2t : t^2 - 1 : t^2 + 1]$. Show that that there is a homeomophism from the line defined by $y = 0$ to X given by

$$[t, 0, 1] \mapsto [2t, t^2 - 1, t^2 + 1]$$

and

$$[1, 0, 0] \mapsto [0, 1, 1].$$