Math V1202. Calculus IV, Section 004, Spring 2007

Practice Midterm 2

Problem 1 Let C be the arc of the parabola $y = \frac{1}{2}x^2$ from $(1, \frac{1}{2})$ to $(2, 2)$. Evaluate the following line integrals.

(a) $\int_C x ds$

(b) $\int_C y dx + x^2 dy$

Problem 2 Consider the vector field

$$\mathbf{F} = (yz\cos(xz))\mathbf{i} + (\sin(xz) - z)\mathbf{j} + (xy\cos(xz) - y)\mathbf{k}.$$

(a) Find a function f such that $\mathbf{F} = \nabla f$.

(b) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve given by

$$\mathbf{r}(t) = \langle (1-t)e^t, t^2, \sin(\frac{\pi}{2}t) \rangle, \quad 0 \leq t \leq 1.$$

Problem 3 Use Green's theorem to evaluate the line integral

$$\oint_C (x^3 - y^3)\ dx + (x^3 + y^3)\ dy$$

where C is the oriented curve shown in Figure 1.

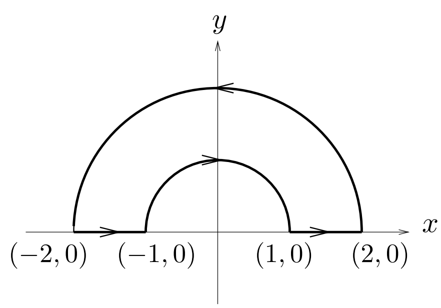

Figure 1: C is the union of two semicircles and two line segments.

Problem 4 Consider the vector field $\mathbf{F} = xyz\,\mathbf{i} + yz\,\mathbf{j} - x^2\,\mathbf{k}$.

(a) Is $\mathbf{F}$ a conservative vector field? Explain.

(b) Is there a vector field $\mathbf{G}$ such that $\operatorname{curl}\mathbf{G} = \mathbf{F}$? Explain.

Problem 5 Let T be the torus with vector equation

$\mathbf{r}(u, v) = (3+\cos u)\cos v\,\mathbf{i} + (3+\cos u)\sin v\,\mathbf{j} + \sin u\,\mathbf{k},\ 0 \leq u \leq 2\pi,\ 0 \leq v \leq 2\pi.$

(a) Calculate $\mathbf{r}_u$, $\mathbf{r}_v$, $\mathbf{r}_u \times \mathbf{r}_v$.

(b) Find an equation for the plane tangent to T at $(\frac{5}{2}, 0, \frac{\sqrt{3}}{2})$.

(c) Find the surface area of T.

Problem 6 Evaluate the surface integral $\iint_S z^4 dS$, where S is the upper hemisphere $x^2 + y^2 + z^2 = 1,\ z \geq 0$.

Problem 7 Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = -y\,\mathbf{i} + x\,\mathbf{j} + z\,\mathbf{k}$, and S is the part of the cone $x^2 + y^2 = z^2$ between the planes $z = 1$ and $z = 4$, with upward orientation.