Math V1202. Calculus IV, Spring 2007, Section 4

Midterm 2

Thursday, March 29, 2007

Name :

INSTRUCTIONS :

1. There are 7 problems on this midterm (total of 12 pages). Please check that your copy contains all of the 12 pages and obtain a new copy of the exam if it does not.

2. Feel free to use the opposite side.

3. In order to receive full credit for any problem, you must show work leading to your answer. Make sure that your final answer is clearly indicated.

4. No calculators, books, notes etc. are allowed.

5. You have 75 minutes to complete this test.

Problem	Possible points	Score
1	18	
2	18	
3	12	
4	10	
5	15	
6	15	
7	12	
Total	100	

Problem 1. Consider a vector field $\mathbf{F}(x, y, z) = \sin y\mathbf{i} + \cos x\mathbf{j} + z^2\mathbf{k}$.

(a) (8%) Is $\mathbf{F}$ conservative? Explain.

Problem 1. (continued)

(b) (10%) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the line segment from $(2, 1, 0)$ to $(0, 1, 3)$.

Problem 2. Consider the vector field

$$\mathbf{F}(x, y, z) = (3x^2yz - 3y)\mathbf{i} + (x^3z - 3x)\mathbf{j} + (x^3y + 2z)\mathbf{k}.$$

(a) (12%) Find a function f such that $\mathbf{F} = \nabla f$.

Problem 2. (continued)

(b) (6%) Use part (a) to evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the oriented curve from $(0, 0, 2)$ to $(0, 1, 0)$ shown in Figure 1.

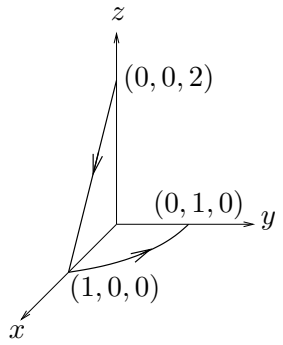

Figure 1: C is the union of the line segment from $(0, 0, 2)$ to $(1, 0, 0)$ and the arc $\{(x, y, z) \mid x^2 + y^2 = 1,\ x \geq 0,\ y \geq 0,\ z = 0\}$ from $(1, 0, 0)$ to $(0, 1, 0)$.

Problem 3. (12%) Use Green's theorem to evaluate the line integral

$$\int_C (\sin(x^3) - y)dx + (x - e^{y^2})dy$$

where C is the rectangle with vertices $(1, 1)$, $(4, 1)$, $(4, 3)$, and $(1, 3)$, oriented counterclockwise.

Problem 4. (10%) Consder two vector fields

$$\mathbf{F}(x,y) = P(x,y)\mathbf{i} + Q(x,y)\mathbf{j}, \quad \mathbf{G}(x,y) = Q(x,y)\mathbf{i} - P(x,y)\mathbf{j}.$$

Show that $\operatorname{curl}\mathbf{F} \cdot \mathbf{k} = \operatorname{div}\mathbf{G}$.

Problem 5. (15%) Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = x\mathbf{i} + y\mathbf{j} - z\mathbf{k}$, and S is the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the plane $z = 0$, with downward orientation.

Problem 6. (15%) Evaluate $\iint_S yz dS$, where S is the part of the sphere $x^2 + y^2 + z^2 = 4$ in the first octant.

Problem 7. (12%) Let S be the part of the cylinder $x^2 + z^2 = R^2$ that lies inside the cylinder $y^2 + z^2 = R^2$. Find the area of S.

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