Mathematics G4403. Modern Geometry **Spring 2012**

Assignment 17

Due Monday, March 5, 2012

[dC] = do Carmo, *Riemannian Geometry*

(1) Let M be a complete Riemannian manifold with non-positive sectional curvature. Prove that

$$|(d\exp_p)_v(w)| \geq |w|$$

for all $p \in M$, all T_pM and all $w \in T_v(T_pM)$.

(2) [dC] page 120-121 Exercise 4. (This is used in the proof on page 223 of [dC].)

(3) [dC] page 238-240 Exercise 5.

(4) Let M be a complete, simply connected Riemannian manifold of constant sectional curvature b. Let $S_\ell(p) \subset M$ be the geodesic sphere with center $p \in M$ and radius $\ell > 0$. Let ν be the inward unit normal to $S_\ell(p)$ at $q \in S_\ell(p)$. Show that for any $x, y \in T_q(S_\ell(p))$,

$$\langle B(x,y), \nu \rangle = \begin{cases} \sqrt{b}\cot(\ell\sqrt{b})\langle x, y\rangle & \text{if } b > 0 \text{ and } \ell < \frac{\pi}{\sqrt{b}}, \\ \frac{1}{\ell}\langle x, y\rangle & \text{if } b = 0, \\ \sqrt{-b}\coth(\ell\sqrt{-b})\langle x, y\rangle & \text{if } b < 0. \end{cases}$$

In particular, if $b > 0$ then $S_{\frac{\pi}{2\sqrt{b}}}(p)$ is a totally geodesic submanifold of M.