

Assignment 14

Due on Monday, February 13, 2012

Notation: [dC] = do Carmo, *Riemannian Geometry*

- (1) Let $f : \mathbb{R}^n - \{p_0\} \rightarrow \mathbb{R}^n - \{p_0\}$ be the inversion

$$f(p) = \frac{p - p_0}{|p - p_0|^2} + p_0.$$

- (a) Let P be a hyperplane in \mathbb{R}^n . Show that if $p_0 \in P$ then $f(P - p_0) = P - \{p_0\}$. Show that if $p_0 \notin P$ then the closure of $f(P)$ is an $(n - 1)$ -sphere passing through p_0 .
- (b) Let S be an $(n - 1)$ -sphere in \mathbb{R}^n . Show that if $p_0 \in S$ then $f(S - \{p_0\})$ is a hyperplane. Show that $f(S)$ is an $(n - 1)$ -sphere if $p_0 \notin S$.
- (2) [dC] page 186 Exercise 9.
- (3) [dC] page 187 Exercise 10.
- (4) [dC] page 188-189, Exercise 12 (b).