

Assignment 16

Due on Monday, February 22, 2010

- (1) Let $f : \mathbb{R}^n - \{p_0\} \rightarrow \mathbb{R}^n - \{p_0\}$ be the inversion

$$f(p) = \frac{p - p_0}{|p - p_0|^2} + p_0.$$

- (a) Let P be a hyperplane in \mathbb{R}^n . Show that if $p_0 \in P$ then $f(P - p_0) = P - \{p_0\}$. Show that if $p_0 \notin P$ then $f(P)$ is an $(n - 1)$ -sphere passing through p_0 .
- (b) Let S be an $(n - 1)$ -sphere in \mathbb{R}^n . Show that if $p_0 \in S$ then $f(S - \{p_0\})$ is a hyperplane. Show that $f(S)$ is an $(n - 1)$ -sphere if $p_0 \notin S$.
- (2) do Carmo page 182-184 Exercise 6, with the additional assumption $n \geq 2$. (Note that there are 5 parts. For part (b), you may assume Exercise 5, which is essentially (1) of Assignment 11. For part (d), see page 177-178.)